Notes on Continuous Phase Modulation (CPM)

References

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2) F. Xiong, “Digital Modulation Techniques”, Artech House 2002, ISBN : 0-89006-970-0, pp. 259-350.

3) T. Aulin, C-E W. Sundberg, “Continuous Phase Modulation-Part I : Full Response”, IEEE Transactions on Communications, Vol. Com. 29, No. 3, March 1981, pp. 196-209.

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1. General Introduction

Continuous phase modulation is used to avoid the abrupt phase changes in PSK or FSK. This way spectrum usage is less. It is based on (analogue) FM principle, where the waveforms with a carrier and without are given by

$$\begin{aligned} u(t) &= \mathrm{Re}\left\{\exp\left[j2\pi f_c t + j\phi(t,\mathbf{a})\right]\right\} = \cos\left[2\pi f_c t + \phi(t,\mathbf{a})\right] \\ s(t) &= \exp\left[j\phi(t,\mathbf{a})\right] \end{aligned} \tag{1.1}$$

where f_c is the carrier frequency. Apart from plotting the waveform to see the smoothing in phase changes (thus causing reductions in bandwidth compared to the conventional PSK) , we will not use the carrier, since its presence does not affect our analysis. So mostly we will deal with $\phi(t,\mathbf{a})$. In such a case, baseband modulated signal will become $s(t)$ as shown on the second line of (1.1).

The term that will cause modulation is $\phi(t,\mathbf{a})$ and will be related to the message signal which is usually taken in the form of ASK (PAM) as follows

$$v(t) = \sum_{-\infty}^{\infty} a_k g_T(t-kT) \tag{1.2}$$

To establish FM in (1.1), we must relate $\phi(t,\mathbf{a})$ to $v(t)$ in the manner $\phi(t,\mathbf{a}) \ \alpha \ \int v(\tau)d\tau$. Now we wish to point out that a_k s are symbols with duration T and taking on values for an *M*ary ASK modulation $\mp 1,\ \mp 3,\ \mp 5,\ \cdots \mp (M-1)$. This way a_k symbols are time independent within a symbol duration of T . And the whole time dependence is embedded in the shaping waveform $g_T(t)$.

$g_T(t)$ will take on different forms, but in all cases we ensure that it is normalized as

$$\int_{-\infty}^{\infty} g_T(t)\, dt = \frac{1}{2} \tag{1.3}$$

Now we examine the different cases of $g_T(t)$

1) Rectangular (LREC)

For this case we define $g_T(t)$ as follows

$$g_T(t) = \begin{cases} \dfrac{1}{2LT} & 0 \le t \le LT \\ 0 & \text{elsewhere} \end{cases} \tag{1.4}$$

In (1.4), *L* is an integer determining over how many symbol durations $g_T(t)$ will extend. In literature, $L=1$ is known as full response, whereas $L>1$ is named partial response. Since $\phi(t,\mathbf{a})$ will be derived from the integral of $v(t)$, we would also be interested in the integral of $g_T(t)$. By taking $L=1$, the integral of $g_T(t)$ which we denote by $q(t)$ will be

$$q(t) = \int_{-\infty}^{t} g_T(\tau)\, d\tau = \begin{cases} \displaystyle\int_0^t \frac{1}{2T}\, d\tau = \frac{t}{2T} & 0 \le t \le T \\ \displaystyle\int_0^T \frac{1}{2T}\, d\tau = \frac{1}{2} & t > T \end{cases} \tag{1.5}$$

The plots of $g_T(t)$ and $q(t)$ are given in Fig. 1.1 for $L=1$.
Now by setting

$$\begin{aligned} \phi(t,\mathbf{a}) &= 4\pi h \int_{-\infty}^{t} v(\tau)\, d\tau \\ &= 2\pi h \sum_{k=-\infty}^{\infty} a_k q(t-kT) \end{aligned} \tag{1.6}$$

h is known as the modulation index. Now consider (1.5) and (as done at the receiver) the symbol interval $\left[kT,\ (k+1)T\right]$ from (1.6)

$$\begin{aligned} \phi(t,\mathbf{a}) &= 2\pi h\left[a_k \frac{(t-kT)}{2T} + \sum_{i=-\infty}^{k-1} 0.5 a_i\right] = \pi h a_k \frac{t}{T} - \pi h k a_k + \pi h \sum_{i=-\infty}^{k-1} a_k \\ &= \pi h a_k \frac{t}{T} + \phi_k \qquad kT \le t \le (k+1)T \quad , \quad \phi_k = \pi h\left(\sum_{i=-\infty}^{k-1} a_i - k a_k\right) \ (\operatorname{mod} 2\pi) \end{aligned} \tag{1.7}$$

Note that in (1.7) $\operatorname{mod} 2\pi$ means that we confine phase changes in the interval 0 to 2π or sometimes $-\pi$ to π. Further note that with $h=0.5$ and $M=2$, $\phi(t,\mathbf{a})$ will be the phase expression for minimum shift keying (MSK).

2) Raised Cosine (LRC)

For an arbitrary L, $g_T(t)$ in this case is

$$g_T(t) = \begin{cases} \dfrac{1}{2LT}\left[1-\cos\left(\dfrac{2\pi t}{LT}\right)\right] & 0 \le t \le LT \\ 0 & \text{elsewhere} \end{cases} \qquad (1.8)$$

The other $g_T(t)$ waveforms are Spectrally Raised Cosine (LSRC), Tamed Frequency Modulation (TFM) and Gaussian Minimum Shift Keying (GMSK).

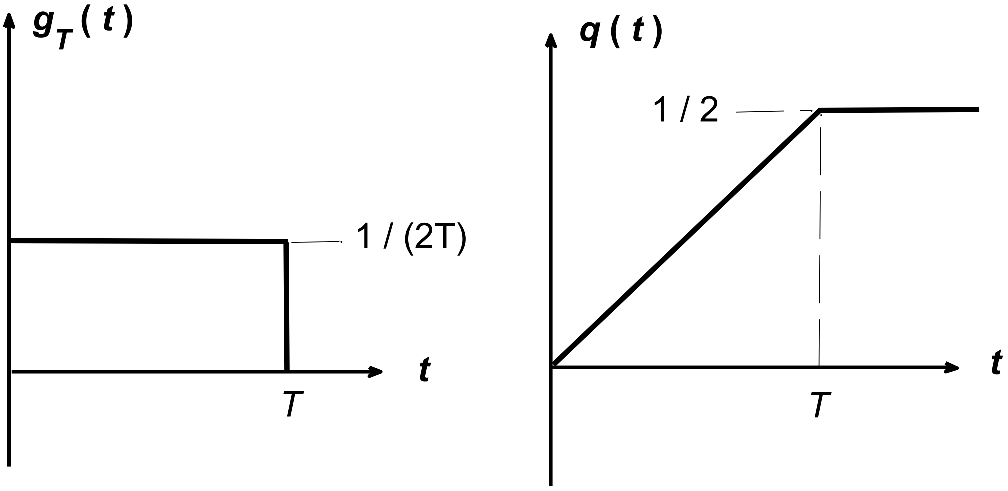

Fig. 1.1 The rectangular shaping waveform $g_T(t)$ and its integral $q(t)$ for $L=1$.

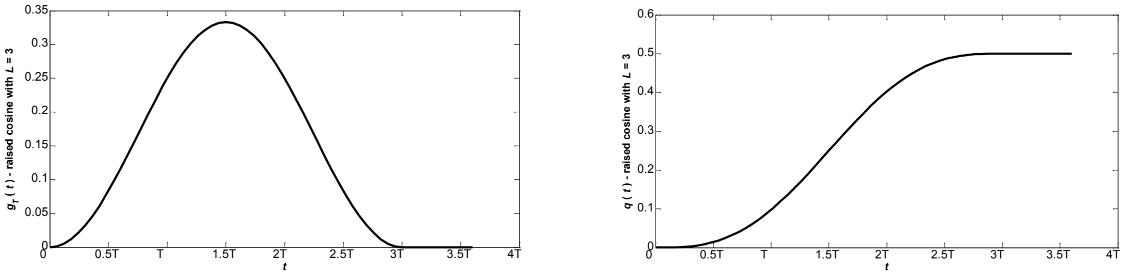

Fig. 1.2 Raised cosine shaping waveform $g_T(t)$ and its integral $q(t)$ for $L = 3$.

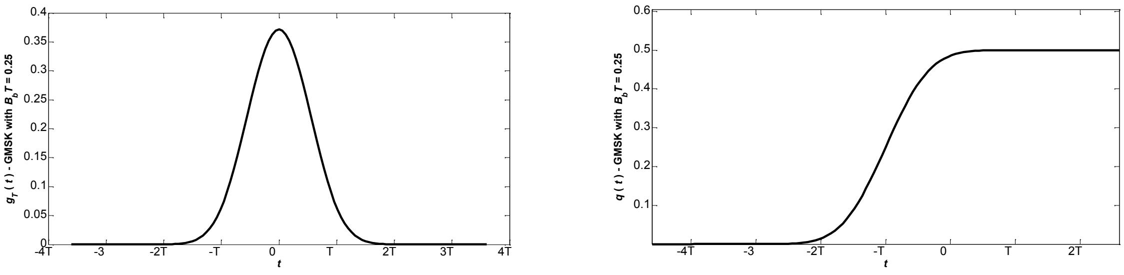

Fig. 1.3 GMSK shaping waveform $g_T(t)$ and its integral $q(t)$ for $B_bT=3$.

2. Phase States of CPM Signals

As seen from the previous section in CPM, the whole matter revolves around the (total) phase term $\phi(t,\mathbf{a})$ which is past dependent as seen from (1.7). This means that CPM has memory and it can be represented in (phase) states and in trellis diagram. To generalize (1.7) for any L, we rearrange it as follows

$$\begin{aligned}&\phi(t,\mathbf{a})=\theta(t,\mathbf{a})+\theta_k\\&\theta(t,\mathbf{a})=2\pi h\sum_{i=k-L+1}^{k}a_iq(t-iT)\quad,\quad\theta_k=\pi h\left(\sum_{i=-\infty}^{k-L}a_i\right)\ (\operatorname{mod}2\pi)\end{aligned}\tag{2.1}$$

The separation in (2.1) implies that $\theta(t,\mathbf{a})$ is the instant phase, representing the changing part of the phase in the time interval $\left[kT,\ (k+1)T\right]$. On the other hand θ_k is the cumulative phase (coming from the past) in the same time interval.

By further analysis, it is possible to simplify the contributing terms of $\theta(t,\mathbf{a})$ and θ_k in $\phi(t,\mathbf{a})$.

If modulation index is a rational and it can be written as $h=2m/p$ where m and p have no common factors and bearing in mind that a_i can only take on values such as $\mp1,\ \mp3,\ \mp5,\ \cdots\mp(M-1)$ then,

$$\begin{aligned}&\theta_k=\frac{2m\pi}{p}\left(\sum_{i=-\infty}^{k-L}a_i\right)=\text{integer multiple of }\frac{2\pi}{p}\\&\text{Number of phase states due to }\theta_k\text{ is }2\pi/(2\pi/p)=p\end{aligned}\tag{2.2}$$

From (2.1) for different values of L, $\theta(t,\mathbf{a})$ can be written as (for the time interval $\left[kT,\ (k+1)T\right]$)

$$\begin{aligned}&\theta(t,\mathbf{a})=2\pi ha_kq(t-kT) && \text{for } L=1\\&\theta(t,\mathbf{a})=2\pi h\left[a_{k-1}q(t-kT+T)+a_kq(t-kT)\right] && \text{for } L=2\\&\theta(t,\mathbf{a})=2\pi h\left[a_{k-2}q(t-kT+2T)+a_{k-1}q(t-kT+T)+a_kq(t-kT)\right] && \text{for } L=3\end{aligned}\tag{2.3}$$

Considering at $t=kT,\ q(t-kT)=0$, the (maximum) number of phase states attained by $\phi(t,\mathbf{a})$ will be the combination of phase states attained by θ_k and $\theta(t,\mathbf{a})$. We express this as $N_{\Phi_k}=pM^{L-1}$ where p is due to cumulative phase, θ_k and M^{L-1} is due to instant phase $\theta(t,\mathbf{a})$. Now at $t=kT$, (2.3) will become

$$\begin{aligned}
&\theta(kT,\mathbf{a}) = 2\pi h a_k q(0) = 0 && \text{for } L=1 \\
&\theta(kT,\mathbf{a}) = 2\pi h a_{k-1} q(T) && \text{for } L=2 \\
&\theta(kT,\mathbf{a}) = 2\pi h\left[a_{k-2} q(2T) + a_{k-1} q(T)\right] && \text{for } L=3
\end{aligned} \tag{2.4}$$

As seen from the first line of (2.4), that is for $L=1$, $\theta(kT,\mathbf{a})$ will always be zero, so the sole contribution to $\phi(t,\mathbf{a})$ will come from θ_k. And we note from (2.1) that θ_k can be computed recursively as shown below

$$\theta_k = \theta_{k-1} + \pi h a_{k-L} \tag{2.5}$$

Now we do a long example by hand first and verify our results by MATLAB and plot the trellis diagrams for CPM.

Example 2.1 : A binary, message signal sequence of $\mathbf{a} = [1,\ -1,\ -1,\ 1,\ -1,\ 1,\ 1,\ 1,\ 1]$, i.e.($M = 2$) is to be continuously phase modulated with $h = 2/3$, find $\phi(t,\mathbf{a})$, when

1. $g_T(t)$ is a rectangular pulse with $L=1$ (named as Example 2.1.1).
2. $g_T(t)$ is a raised cosine pulse with $L=3$ (named as Example 2.1.2).

For each case draw the state diagrams, trellis diagrams and the trellis path traced by the given message signal sequence $\mathbf{a}$. Verify your results in MATLAB.

Solution for Example 2.1.1 : We envisage the following *k* increments, time axis and the indexing for message signal sequence $\mathbf{a}$

$$\begin{aligned}
\mathbf{a} &= [1,\ -1,\ -1,\ 1,\ -1,\ 1,\ 1,\ 1,\ 1] \\
&= [a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8] \\
\mathbf{k} &= [0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8] \\
\mathbf{t} &= [0,\ T,\ 2T,\ 3T,\ 4T,\ 5T,\ 6T,\ 7T,\ 8T]
\end{aligned} \tag{2.6}$$

In the case of rectangular $g_T(t)$ with $L=1$, for $\theta(t,\mathbf{a})$, we use the upper line of 2.4, thus

$$\theta(0,\mathbf{a}) = \theta(kT,\mathbf{a}) = 2\pi h a_k q(0)\ ,\ q(0)=0\ ,\ \theta(0,\mathbf{a})=0 \tag{2.7}$$

Note that (2.7) is valid for any k including $k=0$, so we shall not repeat the calculation of $\theta(kT,\mathbf{a})$ for other k s. For θ_k, we can use (2.5). In order to start with zero phase at $t=0$ (or $k=0$), it must be that $\theta_0 = 0$.

So from (2.1), we get

$$\phi(0,\mathbf{a}) = \theta(0,\mathbf{a}) + \theta_0 = 0 \tag{2.8}$$

Now we go to the beginning of next time slot in the time interval of $\left[kT,\ (k+1)T\right]$, where we make $k=1$. We use (2.5) and find that

$$\theta_1=\theta_0+\pi h a_0=0+\pi\frac{2}{3}\times 1=\frac{2\pi}{3} \tag{2.9}$$

So $\phi(T,\mathbf{a})$ is found to be

$$\phi(T,\mathbf{a})=\theta(T,\mathbf{a})+\theta_1=\frac{2\pi}{3} \tag{2.10}$$

For the subsequent $\phi(t,\mathbf{a})$, we do not show the details, but write the results directly. It is clear that in the subsequent, $\theta(t,\mathbf{a})$ (instant phase) will be zero similar to (2.9) and (2.10), hence the sole contribution to $\phi(t,\mathbf{a})$ will come from θ_k (accumulated phase).

$$\begin{aligned}
&\text{At } t=2T \text{ or } k=2\ ,\ \theta_2=0\quad ,\quad \text{so } \phi(2T,\mathbf{a})=0\\
&\text{At } t=3T \text{ or } k=3\ ,\ \theta_3=-\frac{2\pi}{3}\quad ,\quad \text{so } \phi(3T,\mathbf{a})=-\frac{2\pi}{3}\\
&\text{At } t=4T \text{ or } k=4\ ,\ \theta_4=0\quad ,\quad \text{so } \phi(4T,\mathbf{a})=0\\
&\text{At } t=5T \text{ or } k=5\ ,\ \theta_5=-\frac{2\pi}{3}\quad ,\quad \text{so } \phi(5T,\mathbf{a})=-\frac{2\pi}{3}\\
&\text{At } t=6T \text{ or } k=6\ ,\ \theta_6=0\quad ,\quad \text{so } \phi(6T,\mathbf{a})=0\\
&\text{At } t=7T \text{ or } k=7\ ,\ \theta_7=\frac{2\pi}{3}\quad ,\quad \text{so } \phi(7T,\mathbf{a})=\frac{2\pi}{3}\\
&\text{At } t=8T \text{ or } k=8\ ,\ \theta_8=\frac{4\pi}{3}\quad ,\quad \text{so } \phi(8T,\mathbf{a})=\frac{4\pi}{3}
\end{aligned} \tag{2.12}$$

(2.12) shows that there are only 3 possible states in the rectangular $g_T(t)$ with $L=1$. Note that the phase of $\frac{4\pi}{3}$ is equivalent to $-\frac{2\pi}{3}$. Therefore the possible states of this case are

$$\begin{aligned}
&\Phi_k=\{\Phi_1,\ \Phi_2,\ \Phi_3\}=\left\{0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right\}\quad \text{if phase space is chosen to be } 0\rightarrow 2\pi\\
&\Phi_k=\left\{0,\ \mp\frac{2\pi}{3}\right\}\qquad \text{if phase space is is chosen to be } -\pi\rightarrow\pi
\end{aligned} \tag{2.13}$$

Shown aligned with respect to the message sequence, our phase sequence will be

$$
\begin{aligned}
\mathbf{a} &= [1,\ -1,\ -1,\ 1,\ -1,\ 1,\ 1,\ 1,\ 1] \\
&= [a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8] \\
\mathbf{k} &= [0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8] \\
\mathbf{t} &= [0,\ T,\ 2T,\ 3T,\ 4T,\ 5T,\ 6T,\ 7T,\ 8T] \\
\phi(t,\mathbf{a}) &= \left[0,\ \frac{2\pi}{3},\ 0,\ \frac{4\pi}{3},\ 0,\ \frac{4\pi}{3},\ 0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right] \ \text{phase space is } 0 \rightarrow 2\pi \\
\phi(t,\mathbf{a}) &= \left[0,\ \frac{2\pi}{3},\ 0,\ -\frac{2\pi}{3},\ 0,\ -\frac{2\pi}{3},\ 0,\ \frac{2\pi}{3},\ -\frac{2\pi}{3}\right] \ \text{phase space is } -\pi \rightarrow \pi
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{a} &= [1,\ -1,\ -1,\ 1,\ -1,\ 1,\ 1,\ 1,\ 1] \\
&= [a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8] \\
&\quad\ \searrow\ \ \searrow\ \ \searrow\ \ \searrow\ \ \searrow\ \ \searrow\ \ \searrow\ \ \searrow\ \ \searrow \\
\phi(t,\mathbf{a}) &= \left[0,\ \frac{2\pi}{3},\ 0,\ \frac{4\pi}{3},\ 0,\ \frac{4\pi}{3},\ 0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right]
\end{aligned}
\tag{2.14}
$$

Running CPMFX_Exp1sur.m together with CPMFX_Exp1.mdl model file produces the following output shown on the first line of (2.15), given along with the theoretically calculated $\theta(t,\mathbf{a})$ (of phase space $0 \rightarrow 2\pi$) form (2.14)

$$
\begin{aligned}
s(t) = \exp[j\phi(t,\mathbf{a})] &= \left[1,\ \frac{-1}{2}+\frac{j\sqrt{3}}{2},\ 1,\ \frac{-1}{2}-\frac{j\sqrt{3}}{2},\ 1,\ \frac{-1}{2}-\frac{j\sqrt{3}}{2},\ 1,\ \frac{-1}{2}+\frac{j\sqrt{3}}{2},\ \frac{-1}{2}-\frac{j\sqrt{3}}{2}\right] \\
\phi(t,\mathbf{a}) &= \left[0,\ \frac{2\pi}{3},\ 0,\ \frac{4\pi}{3},\ 0,\ \frac{4\pi}{3},\ 0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right] \ 0 \rightarrow 2\pi
\end{aligned}
\tag{2.15}
$$

As seen from (2.15), there perfect agreement between the two lines of (2.15). CPMFX_Exp1.mdl model file also produces a scope output which is displayed in Fig. 2.1. Again this figure is also in agreement with (2.15).

If we want to observe the (smooth) progression of changes in the phase, $\phi(t,\mathbf{a})$, then in CPMFX_Exp1sur.m, we substitute a value greater than unity for the parameter sPS. For instance if we set this parameter to 32, we obtain the illustration given in Fig. 2.2. To verify the curves in Fig. 2.2, we consider (1.5), (2.3), (2.4) and Fig. 1.1. Accordingly we see that during the interval of $kT < t < (k+1)T$ (note that we have excluded the end points at $t = kT,\ t = (k+1)T$), the time dependence of $\phi(t,\mathbf{a})$ is solely embedded in $\theta(t,\mathbf{a})$, θ_k contributes nothing. In that interval $\phi(t,\mathbf{a})$ will be given by

$$
\theta(t,\mathbf{a}) = 2\pi h a_k q(t-kT) = \frac{2\pi t}{3T} \quad \text{assuming} \quad a_k = 1 \tag{2.16}
$$

This way the phase will follow a trajectory the form of

$$s(t) = \exp\left[j\phi(t,\mathbf{a})\right] = \exp\left[j\frac{2\pi t}{3T}\right] \tag{2.17}$$

(2.17) is plotted in Fig. 2.2, where we see that the end points, i.e. $t = kT$ agree perfectly with the ones listed in (2.15) and Fig. 2.1. It is important to realize that, the smooth phase trajectory of Fig. 2.1 will be used in the construction of CPM waveform with carrier, i.e. $u(t)$. We show $u(t)$ of this example in Fig. 2.3. As seen, there are no abrupt phase (frequency) changes in $u(t)$ like the case of conventional PSK. Note that this illustration of $u(t)$ is somewhat exaggerated, in practice, there are many cycles of the carrier within one symbol duration.

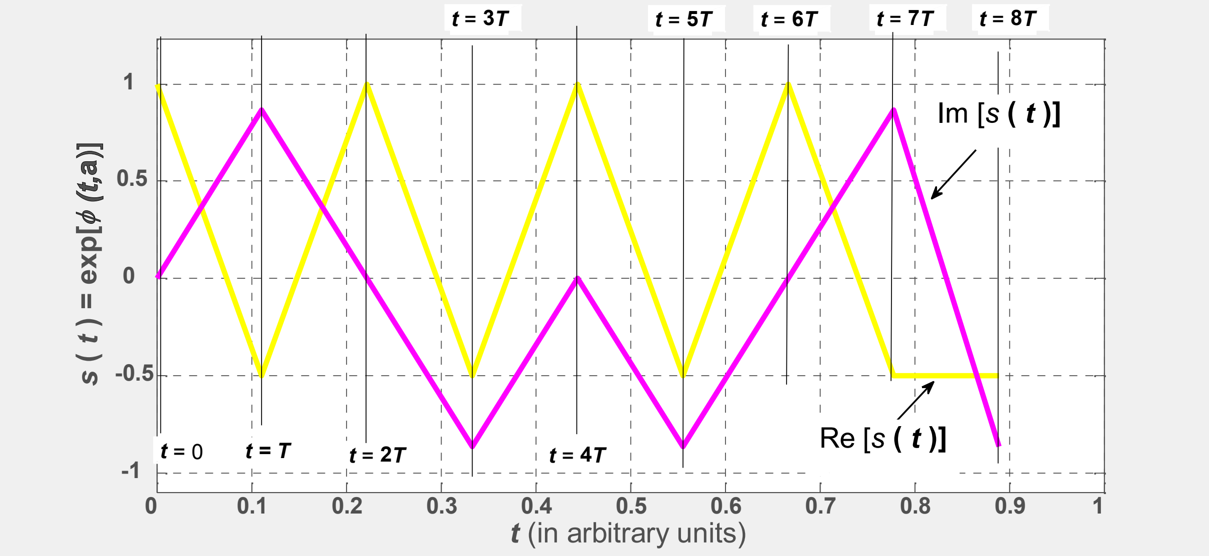

Fig. 2.1 The scope output from CPMFX_Exp1.mdl for a rectangular pulse of $g_T(t)$ with $L = 1$ in Example 2.1.1

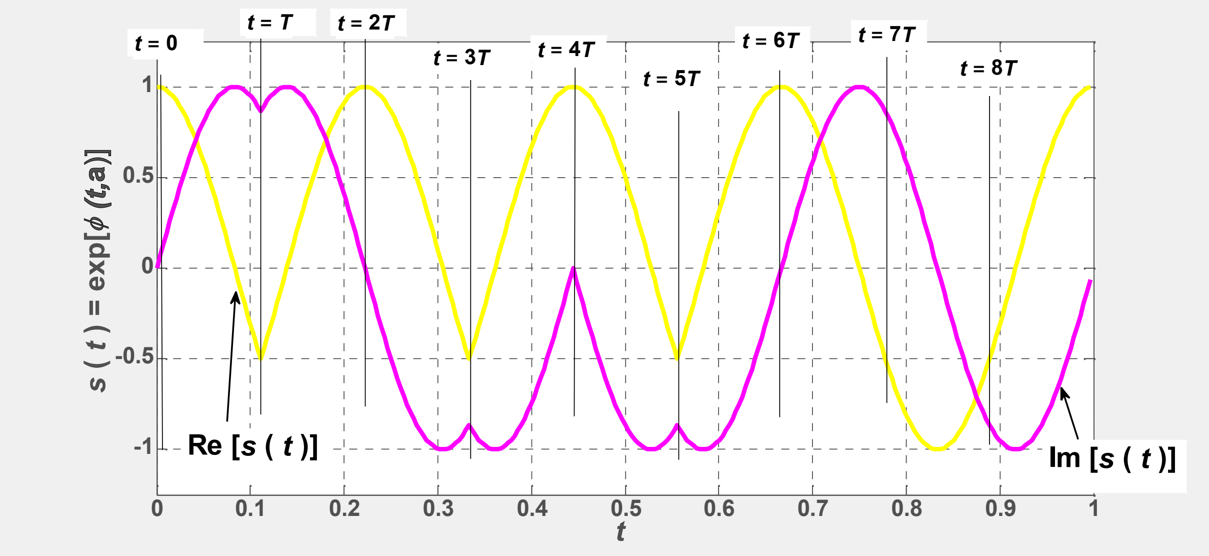

Fig. 2.2 The scope output from CPMFX_Exp1.mdl for a rectangular pulse of $g_T(t)$ with $L = 1$ in Example 2.1.1 with a smooth phase trajectory.

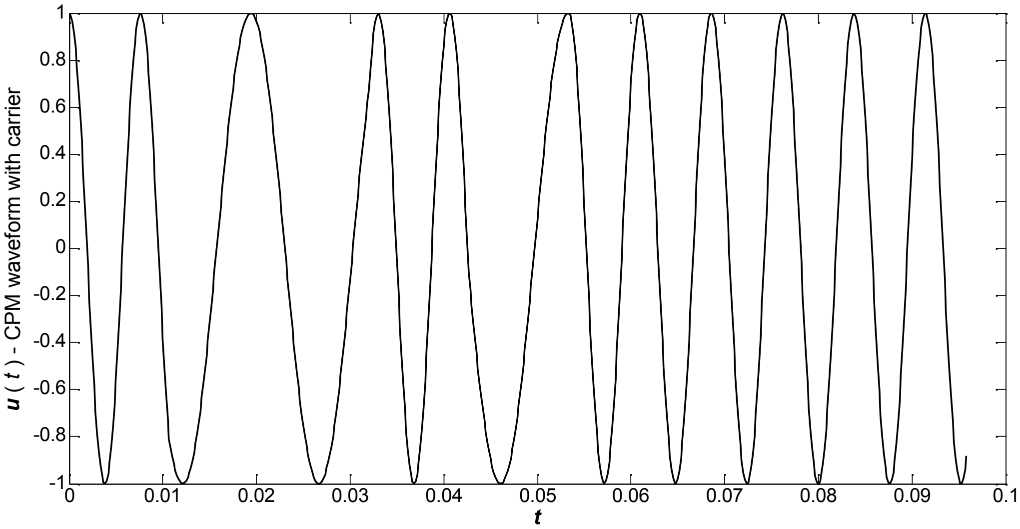

Fig. 2.3 The scope output from CPMFX_Exp1.mdl for a rectangular pulse of $g_T(t)$ with $L=1$ in Example 2.1.1 with a smooth phase trajectory.

Now we construct the state trellis diagram of the case $g_T(t)$ being a rectangular pulse with $L=1$. From the discussion regarding p in (2.2) and also from (2.13), we know that in this case number of distinct phase states are 3, these are $\Phi_{\mathrm{k}} = \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ if phase space is chosen to be $0 \to 2\pi$. Changes between these states will occur depending on a_{k-1} being -1 or +1.

The related state trellis diagram including some of the initial transitions up to a_2 can be found in Fig. 2.4. For this figure, it is important to emphasize once again that in the case of $L=1$, the (total) phase $\phi(t,\mathbf{a})$ is based on the cumulative phase θ_k, with no contribution coming from the instant phase, $\theta(t,\mathbf{a})$. Hence we could have equally denoted the phase states of Fig. 2.5, by θ_k instead of Φ_{k}.

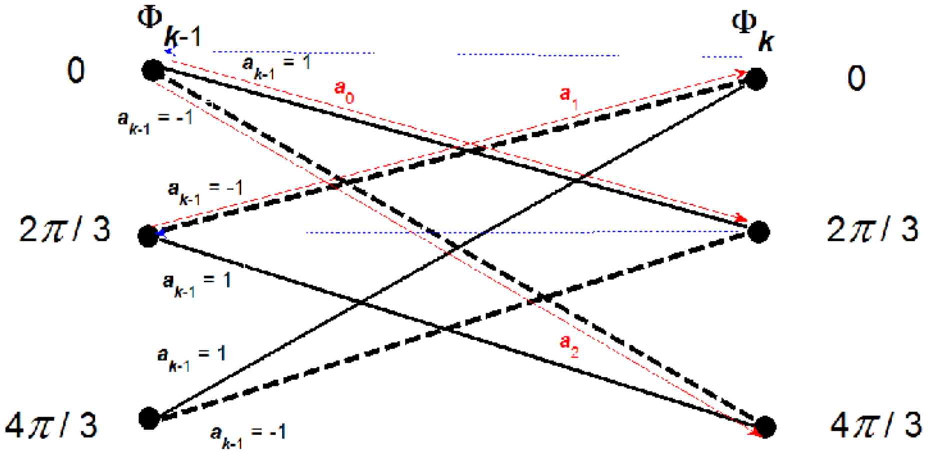

Fig. 2.4 State trellis diagram for rectangular pulse of $g_T(t)$ with $L=1$ in Example 2.1.1

Exercise 2.1 : Draw the trellis signal path of $\mathbf{a} = [1,\ -1,\ -1,\ 1,\ -1,\ 1,\ 1,\ 1,\ 1]$ against time. Use the same notation as that of Fig. 2.4.

Exercise 2.2 : Without changing the other parameters, run CPMFX_Exp1sur.m for a message sequence of $\mathbf{a} = [1,\ -1,\ -1,\ -1,\ -1,\ 1,\ 1,\ 1,\ -1]$ and find $\phi(t,\mathbf{a})$ and $s(t)$, observe the scope output. Verify that you find the same result for $\phi(t,\mathbf{a})$ by hand. Plot the trellis signal path against time, by the same notation as that of Fig. 2.3.

Exercise 2.3 : Repeat exercise 2.2 for $h = 3/4$, $h = 4/7$ and find $\phi(t,\mathbf{a})$ and $s(t)$,both by running CPMFX_Exp1sur.m and by hand. Comment what differences you see, compared with the case of $h = 2/3$.

Solution for Example 2.1.2 : Now we go on to more sophisticated case of $g_T(t)$ being a raised cosine pulse with $L = 3$. Here we note that the binary symbols, a_k prior to a_0 had no effect on $\theta(t,\mathbf{a})$ in the previous case when $L = 1$. Their effect was accumulated in θ_0 making the starting phase in $\phi(t = 0,\mathbf{a})$ zero. But when $L = 3$, the symbols a_{-1} and a_{-2} have to be taken into account as seen from the third lines of (2.3) and (2.4). So initially we must determine what θ_0 should be, so that condition $\phi(t = 0,\mathbf{a}) = 0$ is satisfied (note that condition is not mandatory but convenient, which means we could have started in any **allowed** phase state of this particular CPM. Matlab also makes such an allowance). So at $t = 0$ (which also means $k = 0$), using the third line of (2.4) and (2.5), we get

$$\phi(0,\mathbf{a}) = \theta(0,\mathbf{a}) + \theta_0 = 2\pi h\left[a_{-2}q(2T) + a_{-1}q(T)\right] + \theta_0 = 0 \qquad (2.18)$$

Now assuming $[a_{-2}\ ,a_{-1}] = [1,\ 1]$ and inserting $q(\ )$ function numeric values from Ref. [2] as $q(2T) = 0.402$, $q(T) = 0.098$ into (2.18), we obtain

$$\begin{aligned}\theta_0 &= -2\pi h\left[a_{-2}q(2T) + a_{-1}q(T)\right] = -2\pi h[0.402 + 0.098] \\ &= -\pi h = -\frac{2\pi}{3} \to \frac{4\pi}{3} \equiv \pi h\left(\sum_{i=-\infty}^{-3} a_i\right)\end{aligned} \qquad (2.19)$$

(2.19) means that in order to have $\phi(t = 0,\mathbf{a}) = 0$, θ_0, which is the accumulated phase from minus infinity up to a_{-3}, must be $-\pi h = -\frac{2\pi}{3} \to \frac{4\pi}{3}\ (\mathrm{mod}\,2\pi \text{ equivalent})$. It is appropriate to clarify an important point here. That is, although, we have assumed that the condition that the total phase $\phi(t,\mathbf{a})$ starting from zero at the time of $t = 0$, can be managed by suitably adjusting θ_0 as shown in (2.19). We know from (2.2) and (2.13) that there are only three allowed phase states for θ_k (for this particular value of the modulation index, $h = 2/3$, these are $\Phi_\mathrm{k} = \left\{0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right\}$). By coincidence such a property is satisfied in (2.19) by the selection of $[a_{-2}\ a_{-1}] = [1,\ 1]$. Note that the same will happen, when we also choose $[a_{-2}\ , a_{-1}] = [-1,\ -1]$. On the other hand settings $[a_{-2}\ , a_{-1}] = [-1,\ 1]$ and $[a_{-2}\ , a_{-1}] = [1,\ -1]$, will also mathematically produce phase values for θ_0 , however these results will not be valid phases for θ_0, since such results violate the condition that θ_k (and consequently θ_0) can only take one of the three phase values from the phase set of $\Phi_{\mathbf{k}} = \left\{0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right\}$. In summary the choice of starting at $\phi(t=0,\mathbf{a}) = 0$ demands the correct and careful selection of symbols prior to $t = 0$. Otherwise, we will encounter misleading and physically meaningless phase values (For further details, see, "Explanation on Fig2_5_Notes on CPM_Sept 2012"). It will be shown later, in some cases, no matter what arrangement is made for prior symbols , it is not possible to start at $\phi(t=0,\mathbf{a}) = 0$, simply because the $\phi(t,\mathbf{a})$ does not have a state of zero phase.

We continue using the third line of (2.4) and the recursive relation in (2.5), hence for $k = 1$ (which also corresponds to $t = T$),

$$\theta(T,\mathbf{a}) = 2\pi h\left[a_{-1}q(2T) + a_0 q(T)\right] = \pi h = \frac{2\pi}{3}\ ,\ \theta_1 = \theta_0 + \pi h a_{-2} = 2\pi \to 0$$

$$\phi(T,\mathbf{a}) = \theta(T,\mathbf{a}) + \theta_1 = \frac{2\pi}{3} \qquad (2.20)$$

Repeating the same for higher k values, we get

$$
\begin{aligned}
&\text{At } t=2T \text{ or } k=2\text{ , } \theta(2T,\mathbf{a})=2\pi h\left[a_0 q(2T)+a_1 q(T)\right]\\
&=2\pi h(0.402-0.098)=0.608\pi h\text{ , } \theta_2=\theta_1+\pi h a_{-1}=\pi h\\
&\text{so } \phi(2T,\mathbf{a})=0.608\pi h+\pi h=1.608\pi h=3.216\pi/3\\
&\text{At } t=3T \text{ or } k=3\text{ , } \theta(3T,\mathbf{a})=2\pi h\left[a_1 q(2T)+a_2 q(T)\right]\\
&=-2\pi h(0.402+0.098)=-\pi h\text{ , } \theta_3=\theta_2+\pi h a_0=2\pi h\\
&\text{so } \phi(3T,\mathbf{a})=-\pi h+2\pi h=\pi h=2\pi/3\\
&\text{At } t=4T \text{ or } k=4\text{ , } \theta(4T,\mathbf{a})=2\pi h\left[a_2 q(2T)+a_3 q(T)\right]\\
&=2\pi h(-0.402+0.098)=-0.608\pi h\text{ , } \theta_4=\theta_3+\pi h a_1=\pi h\\
&\text{so } \phi(4T,\mathbf{a})=-0.608\pi h+\pi h=0.392\pi h=0.784\pi/3\\
&\text{At } t=5T \text{ or } k=5\text{ , } \theta(5T,\mathbf{a})=2\pi h\left[a_3 q(2T)+a_4 q(T)\right]\\
&=2\pi h(0.402-0.098)=0.608\pi h\text{ , } \theta_5=\theta_4+\pi h a_2=0\\
&\text{so } \phi(5T,\mathbf{a})=0.608\pi h=1.216\pi/3\\
&\text{At } t=6T \text{ or } k=6\text{ , } \theta(6T,\mathbf{a})=2\pi h\left[a_4 q(2T)+a_5 q(T)\right]\\
&=2\pi h(-0.402+0.098)=-0.608\pi h\text{ , } \theta_6=\theta_5+\pi h a_3=\pi h\\
&\text{so } \phi(6T,\mathbf{a})=0.392\pi h=0.784\pi/3\\
&\text{At } t=7T \text{ or } k=7\text{ , } \theta(7T,\mathbf{a})=2\pi h\left[a_5 q(2T)+a_6 q(T)\right]\\
&=2\pi h(0.402+0.098)=\pi h\text{ , } \theta_7=\theta_6+\pi h a_4=0\\
&\text{so } \phi(7T,\mathbf{a})=\pi h=2\pi/3\\
&\text{At } t=8T \text{ or } k=8\text{ , } \theta(8T,\mathbf{a})=2\pi h\left[a_6 q(2T)+a_7 q(T)\right]\\
&=2\pi h(0.402+0.098)=\pi h\text{ , } \theta_8=\theta_7+\pi h a_5=\pi h\\
&\text{so } \phi(8T,\mathbf{a})=2\pi h=4\pi/3
\end{aligned}
\tag{2.21}
$$

Similar to (2.14), from (2.18) to (2.21), we can construct the following for the case of $g_T(t)$ being a raised cosine pulse with $L=3$ as shown below in (2.22). An important note is that, the phases, $\phi(t,\mathbf{a})$ could have also been calculated using the second line of (1.6).

$$
\begin{aligned}
\mathbf{a}&=[1,\ -1,\ -1,\ 1,\ -1,\ 1,\ 1,\ 1,\ 1]\\
&=[a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8]\\
\mathbf{k}&=[0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8]\\
\mathbf{t}&=[0,\ T,\ 2T,\ 3T,\ 4T,\ 5T,\ 6T,\ 7T,\ 8T]\\
\phi(t,\mathbf{a})&=\left[0,\ \frac{2\pi}{3},\ \frac{3.218\pi}{3},\ \frac{2\pi}{3},\ \frac{0.782\pi}{3},\ \frac{1.216\pi}{3},\ \frac{0.782\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right]
\end{aligned}
\tag{2.22}
$$

The result for $\phi(t,\mathbf{a})$ in (2.22) can be verified again by running CPMFX_Exp1sur.m with the settings of $g_T(t)$ being a raised cosine pulse with $L=3$. (2.22) shows that there are 6 distinct phase states, which are

$$\Phi_{\mathbf{k}} = \left\{0,\ \frac{0.782\pi}{3},\ \frac{1.216\pi}{3},\ \frac{2\pi}{3},\ \frac{3.216\pi}{3},\ \frac{4\pi}{3}\right\} \tag{2.23}$$

On the other hand, according to the rule derived above, there could as many as $N_{\Phi_k} = pM^{L-1} = 12$. To find all possible states, from (2.2) we know that number of possible phase states due to θ_k is obtained from integer multiples of $\frac{2\pi}{p}$ (where is the denominator is the denominator of the modulation index, $h = 2/3 = 2m/p$) and these are listed below

$$\Phi_{\mathbf{k}}(\theta_k) = \left\{0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right\} \tag{2.24}$$

The dependence of $\theta(kT,\mathbf{a}) = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right]$ on the past two symbols a_{k-2} and a_{k-1} will produce four additional phase states. Hence all together, there will be twelve phase states of $\phi(t,\mathbf{a})$, as calculated and shown below

$$\begin{aligned}
&\Phi_1 : [a_{k-2}\ , a_{k-1}] = [-1,-1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = 4.1888\\
&\Phi_2 : [a_{k-2}\ , a_{k-1}] = [+1,-1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = 1.2755\\
&\Phi_3 : [a_{k-2}\ , a_{k-1}] = [-1,+1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = 5.0077\\
&\Phi_4 : [a_{k-2}\ , a_{k-1}] = [+1,+1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = 2.0944\\
&\Phi_5 : [a_{k-2}\ , a_{k-1}] = [-1,-1]\ ,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 2\pi/3 = 6.2832\\
&\Phi_6 : [a_{k-2}\ , a_{k-1}] = [+1,-1]\ ,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 2\pi/3 = 3.3699\\
&\Phi_7 : [a_{k-2}\ , a_{k-1}] = [-1,+1]\ ,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 2\pi/3 = 0.8189\\
&\Phi_8 : [a_{k-2}\ , a_{k-1}] = [+1,+1]\ ,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 2\pi/3 = 4.1888\\
&\Phi_9 : [a_{k-2}\ , a_{k-1}] = [-1,-1]\ ,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 4\pi/3 = 2.0944\\
&\Phi_{10} : [a_{k-2}\ , a_{k-1}] = [+1,-1]\ ,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 4\pi/3 = 5.4643\\
&\Phi_{11} : [a_{k-2}\ , a_{k-1}] = [-1,+1]\ ,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 4\pi/3 = 2.9133\\
&\Phi_{12} : [a_{k-2}\ , a_{k-1}] = [+1,+1]\ ,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 4\pi/3 = 6.2832
\end{aligned} \tag{2.25}$$

As seen from (2.25), several phase states are identical, namely, Φ_1 and Φ_8, Φ_4 and Φ_9, Φ_5 and Φ_{12}. This means that in (2.25), there are only nine distinct states. Rearranging in this manner, converting phase values into a fraction of $\pi/3$ in an ascending order, we get

$$\begin{aligned}
\Phi_k &= \{\Phi_5 \text{ and } \Phi_{12}\,,\ \Phi_7\,,\ \Phi_2\,,\ \Phi_4 \text{ and } \Phi_9\,,\ \Phi_{11}\,,\ \Phi_6\,,\ \Phi_1 \text{ and } \Phi_8\,,\ \Phi_3\,,\ \Phi_{10}\} \\
\Phi_k &= \{0\,,\ 0.8189\,,\ 1.2755\ \ 2.0944\,,\ 2.9133\,,\ 3.3699\,,\ 4.1888\,,\ 5.0077\,,\ 5.4643\} \\
\Phi_k &= \left\{0\,,\ \frac{0.782\pi}{3}\,,\ \frac{1.218\pi}{3}\,,\ \frac{2\pi}{3}\,,\ \frac{2.782\pi}{3}\,,\ \frac{3.218\pi}{3}\,,\ \frac{4\pi}{3}\,,\ \frac{4.782\pi}{3}\,,\ \frac{5.218\pi}{3}\right\}
\end{aligned} \tag{2.26}$$

So as an exercise run CPMFX_Exp1sur.m with the settings of $[a_{-2}\,, a_{-1}] = [1,\ 1]$ or $[a_{-2}\,, a_{-1}] = [-1,\ -1]$ with $L = 3\,, M = 2$ for different arrangements input message signal (vt) to see if you can generate all distinct states of (2.26).

Now we come to draw the trellis diagram of the above case, i.e. the raised cosine pulse with $L = 3$. Compared to the case of $L = 1$, there are more phase states in $L = 3$. Furthermore, the contribution to the (total) phase of $\phi(t, \mathbf{a})$ from the cumulative phase of $\theta(t, \mathbf{a})$ is no longer zero. Therefore here, the trellis diagram will be different. It cannot be solely based on distinct phase states given in (2.26), but must embrace all twelve states of (2.25). Such a trellis diagram employing the notational arrangement of (2.25) is shown in Fig. 2.5.

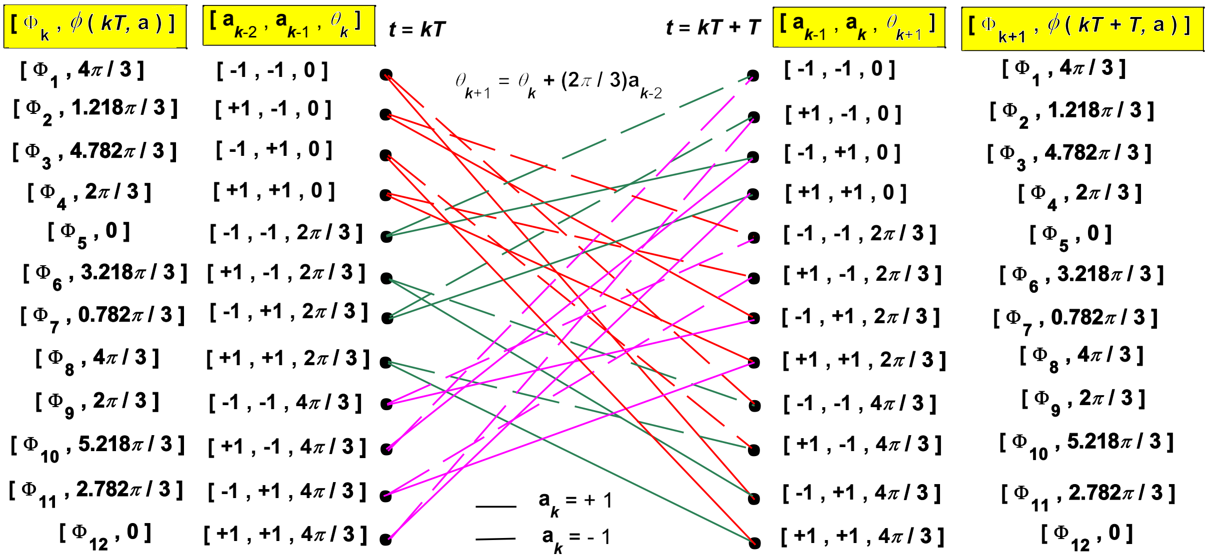

Fig. 2.5 State trellis diagram for raised cosine pulse of $g_T(t)$ with $L = 3$ in Example 2.1.2. Derived from (2.25) and (2.26).

Exercise 2.5 : Verify the phase states in (2.25) and (2.26) and the state trellis diagram given in Fig. 2.5 by running CPMFX_Exp1sur.m with the appropriate parameter settings.

Example 2.2 : By, and keeping the same settings of Example 2.1.2, but changing the pulse length of $g_T(t)$ to $L = 2$, find the output of the CPM modulator, i.e., $\phi(t, \mathbf{a})$ and both by running CPMFX_Exp1sur.m and by hand.

Solution for Example 2.2 : From the rule of $N_{\Phi_k} = pM^{L-1}$, since $p = 3$ we have three phase states due to the cumulative phase term, θ_k and from instant phase term, $\theta(t, \mathbf{a})$, we have two states since $M^{L-1} = 2^1 = 2$. All together, the number of (nondistinct) phases attained by the (total) phase term,

$\phi(t,\mathbf{a})$ will be six. It is possible to calculate the phases like in (2.25), but this time we have a dependence only on the symbol a_{k-1}. Note that from (1.8) and (1.5), for the raised cosine shaping waveform, we find that $q(T)=0.25$. The results are given in (2.27) (again confined to the $0 \rightarrow 2\pi$ interval)

$$\begin{aligned}
&\Phi_1 : a_{k-1}=-1\,,\ \theta_k=0,\ \phi(t,\mathbf{a})=\theta(kT,\mathbf{a})+\theta_k=2\pi h a_{k-1}q(T)+0=5.236 && = 5\pi/3\\
&\Phi_2 : a_{k-1}=+1\,,\ \theta_k=0,\ \phi(t,\mathbf{a})=\theta(kT,\mathbf{a})+\theta_k=2\pi h a_{k-1}q(T)+0=1.0472 && = \pi/3\\
&\Phi_3 : a_{k-1}=-1\,,\ \theta_k=2\pi/3,\ \phi(t,\mathbf{a})=\theta(kT,\mathbf{a})+\theta_k=2\pi h a_{k-1}q(T)+2\pi/3=1.0472 && = \pi/3\\
&\Phi_4 : a_{k-1}=+1\,,\ \theta_k=2\pi/3,\ \phi(t,\mathbf{a})=\theta(kT,\mathbf{a})+\theta_k=2\pi h a_{k-1}q(T)+2\pi/3=3.1416 && = \pi\\
&\Phi_5 : a_{k-1}=-1\,,\ \theta_k=4\pi/3,\ \phi(t,\mathbf{a})=\theta(kT,\mathbf{a})+\theta_k=2\pi h a_{k-1}q(T)+4\pi/3=3.1416 && = \pi\\
&\Phi_6 : a_{k-1}=+1\,,\ \theta_k=4\pi/3,\ \phi(t,\mathbf{a})=\theta(kT,\mathbf{a})+\theta_k=2\pi h a_{k-1}q(T)+4\pi/3=5.236 && = 5\pi/3\\
&\text{Distinct phase states} \qquad \Phi_k=\left\{\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}\right\} && (2.27)
\end{aligned}$$

As seen from (2.27), in this particular case, there are only three distinct phase states. The interesting thing about the chosen example is that it does not include a zero phase state. So it cannot start with zero phase at $t=0$. A note about the MATLAB implementation is in order. In the CPM Modulator Baseband block which is inserted into the model file CPMFx_Exp1.mdl that we are using, there is the Phase Offset setting that corresponds to the phase at the instance of $t=0$. If this Phase Offset setting is made zero (which is not possible according to (2.27)), then by running the m file CPMFx_Exp1.m, we will get the distinct (total) phase values, $\theta(kT,\mathbf{a})$ as $\Phi_k=\left\{0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}\right\}$ which does not of course agree with the last line of (2.27). In order to get the correct phase values, that is the ones listed in (2.27), all we have to do is to set the Phase Offset (written as phaseOffset in CPMFx_Exp1.m) to one of actually possible the phase states, i.e., $\Phi_k=\left\{\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}\right\}$.

Now we show the state trellis diagram of the case of $L=2$ in Fig. 2.6.

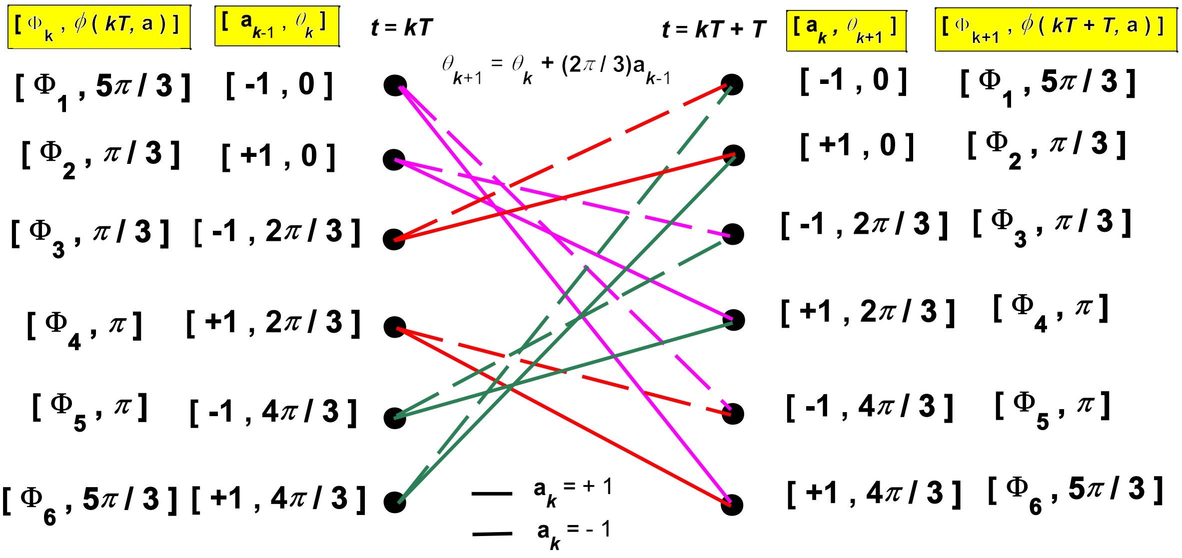

Fig. 2.6 State trellis diagram for raised cosine pulse of $g_T(t)$ with $L=2$ in Example 2. 2. Derived from (2.27).

Exercise 2.6 : Verify the phase states in (2.27) and the state trellis diagram given in Fig. 2.6 by running CPMFX_Exp1sur.m with the appropriate parameter settings.

Example 2.3 : Now, we examine a case of $M = 4$ and keep the other parameters the same as Example 2. 2.

Solution for Example 2.3 : Again using the rule of $N_{\Phi_k} = pM^{L-1}$, since $p = 3$ we have three phase states due to the cumulative phase term, θ_k and from instant phase term, $\theta(t,\mathbf{a})$, we have four states since $M^{L-1} = 4^1 = 4$. All together, the number of (nondistinct) phases attained by the (total) phase term, $\phi(t,\mathbf{a})$ will be twelve.

Observing the guidelines given for (2.25) and (2.27), we perform the phase calculations of this case as follows

$$\begin{array}{ll}
 & \text{Confined to interval } 0 \rightarrow 2\pi \\
\Phi_1 : a_{k-1} = -3\,,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = -3.1416 & = \pi \\
\Phi_2 : a_{k-1} = -1\,,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = -1.0472 & = 5\pi/3 \\
\Phi_3 : a_{k-1} = +1\,,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = 1.0472 & = \pi/3 \\
\Phi_4 : a_{k-1} = +3\,,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = 3.1416 & = \pi \\
\Phi_5 : a_{k-1} = -3\,,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 2\pi/3 = -1.0472 & = 5\pi/3 \\
\Phi_6 : a_{k-1} = -1\,,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 2\pi/3 = 1.0472 & = \pi/3 \\
\Phi_7 : a_{k-1} = +1\,,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 2\pi/3 = 3.1416 & = \pi \\
\Phi_8 : a_{k-1} = +3\,,\ \theta_k = 2\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 2\pi/3 = 5.236 & = 5\pi/3 \\
\Phi_9 : a_{k-1} = -3\,,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 4\pi/3 = 1.0472 & = \pi/3 \\
\Phi_{10} : a_{k-1} = -1\,,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 4\pi/3 = 3.1416 & = \pi \\
\Phi_{11} : a_{k-1} = +1\,,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 4\pi/3 = 5.236 & = 5\pi/3 \\
\Phi_{12} : a_{k-1} = +3\,,\ \theta_k = 4\pi/3,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 4\pi/3 = 7.3304 & = \pi/3
\end{array}$$

$$\text{Distinct phase states} \qquad \Phi_{\mathbf{k}} = \left\{ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3} \right\} \tag{2.28}$$

It is interesting to note that distinct phase states of this case are exactly the same as Example 2.2, where $L = 2,\ M = 2$, which means $M = 4$ has no role in determining the distinct phases.

The state trellis diagram of this example is depicted in Fig. 2.7.

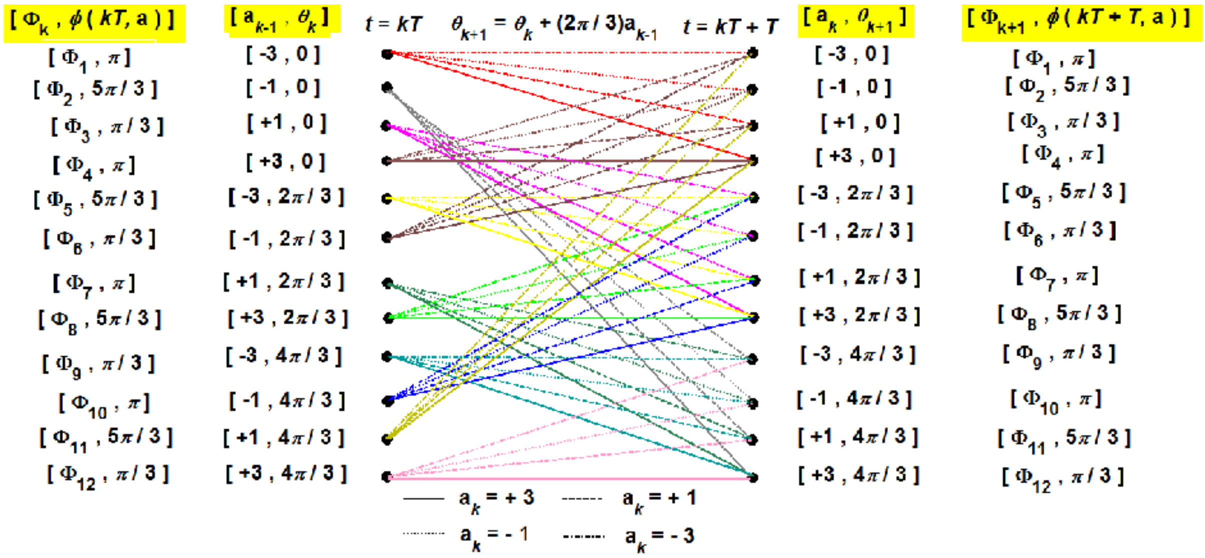

Fig. 2.7 State trellis diagram for raised cosine pulse of $g_T(t)$ with $L = 2$ and $M = 4$ in Example 2. 3. Derived from (2.28).

Exercise 2.7 : Verify the phase states in (2.28) and the state trellis diagram given in Fig. 2.7 by running CPMFX_Exp1sur.m with the appropriate parameter settings.

3. Estimation of Probability of Error for CPM Signals

Initially we start with a test case that our CPM modulator and demodulator will function correctly, when modulator is connected directly to demodulator, i.e. no AWGN channel is used inbetween. For this purpose, CPMFXC_Exp1sur_test.mdl, CPMFXC_Exp1_test.m and sGen.mdl MATLAB files are prepared. Here the aim is get zero error for a given set of M, h, L, shaping waveform (gtshape), samples per second (sPS), trace back length (tBL) values. This is calculated by the symerr function on the last two lines of CPMFXC_Exp1_test.m. Although not necessary, this calculation also entails the PSK comparison case. It is found that, if necessary at $M > 2$, errors are eliminated by choosing $\mathrm{sPS} > 1$, while larger values of tBL will reduce errors toward zero.

The actual MATLAB files that are used to obtain the probability of error curves of CPM against an equivalent PSK (on the same M and equal average signal energy basis) are GDModulator.mdl, GModulator.mdl, MaryGen.mdl, PeCPMsur.m where the last m file is run to obtain results.

Exercise 3.1 : By testing your parameters first by the MATLAB file set, CPMFXC_Exp1sur_test.mdl, CPMFXC_Exp1_test.m and sGen.mdl, then using the actual MATLAB files of GDModulator.mdl, GModulator.mdl, MaryGen.mdl, PeCPMsur.m, find the dependence of probability error of CPM on M, h, L. Also record the probability of error advantages of CPM against PSK and QAM.