Explanations (Tracing) on Fig. 2.5 of Notes on CPM – 09.12.2014 –

Revised on 07.12.2017

$$\theta(kT,\mathbf{a}) = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] \quad \text{instant phase}$$

$$\theta_k = \theta_{k-1} + \pi h a_{k-3} \quad \text{cumulative phase}$$

$$\phi(t,\mathbf{a}) = \theta(t,\mathbf{a}) + \theta_k = \theta(kT,\mathbf{a}) + \theta_k \quad \text{total phase} \tag{1}$$

$$\begin{aligned}
\mathbf{a} &= [1, \ -1, \ -1, \ 1, \ -1, \ 1, \ 1, \ 1, \ 1] \\
&= [a_0, \ a_1, \ a_2, \ a_3, \ a_4, \ a_5, \ a_6, \ a_7, \ a_8] \\
\mathbf{k} &= [0, \ 1, \ 2, \ 3, \ 4, \ 5, \ 6, \ 7, \ 8] \\
\mathbf{t} &= [0, \ T, \ 2T, \ 3T, \ 4T, \ 5T, \ 6T, \ 7T, \ 8T] \\
\phi(t,\mathbf{a}) &= \left[0, \ \frac{2\pi}{3}, \ \frac{3.218\pi}{3}, \ \frac{2\pi}{3}, \ \frac{0.782\pi}{3}, \ \frac{1.216\pi}{3}, \ \frac{0.782\pi}{3}, \ \frac{2\pi}{3}, \ \frac{4\pi}{3}\right]
\end{aligned} \tag{2}$$

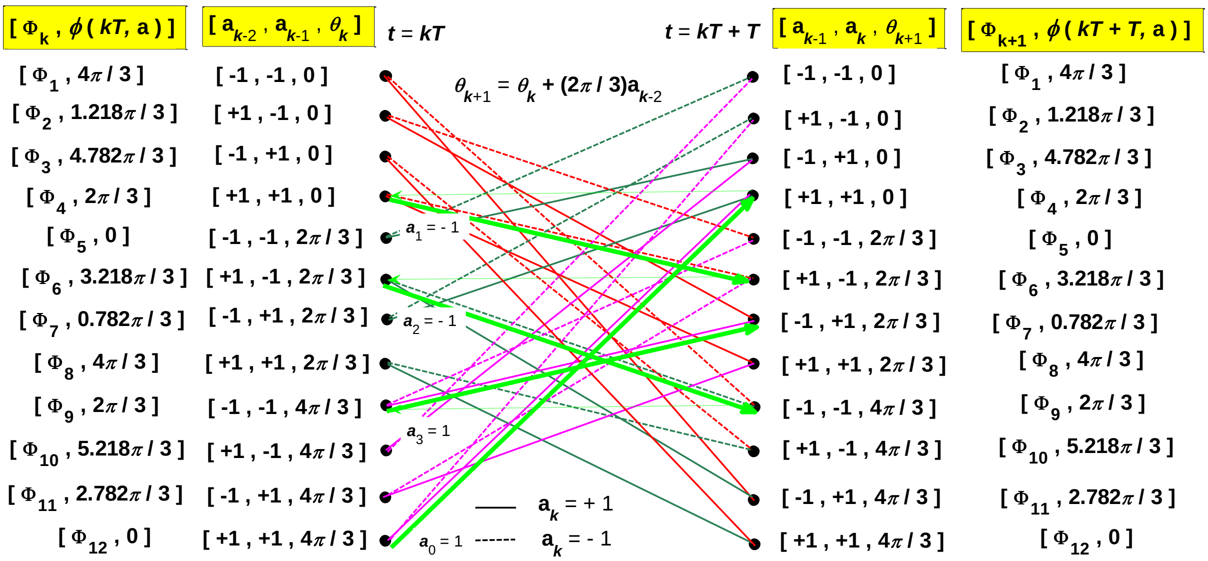

Fig. 2.5 State trellis diagram for raised cosine pulse of $g_T(t)$ with $L = 3$ in Example 2.1.2. Derived from (2.25) and (2.26). Green lines show the tracing for the input message signal sequence of

$$\begin{aligned}
\mathbf{a} &= [1, \ -1, \ -1, \ 1] \\
&= [a_0, \ a_1, \ a_2, \ a_3] \\
\mathbf{k} &= [0, \ 1, \ 2, \ 3] \\
\mathbf{t} &= [0, \ T, \ 2T, \ 3T] \\
\phi(t,\mathbf{a}) &= \left[0, \ \frac{2\pi}{3}, \ \frac{3.218\pi}{3}, \ \frac{2\pi}{3}, \ \frac{0.782\pi}{3}\right]
\end{aligned} \tag{3}$$

Note that the interpretation in (3) is slightly different from (2).

----

1) Sample calculations for the transitions from Φ_{12} , 0 to Φ_4 , $2\pi/3$ and Φ_2 , $1.218\pi/3$

By setting $[a_{-2}\ , a_{-1}] = [a_{k-2}\ , a_{k-1}] = [1,\ 1]$, we find, as explained in (2.18) and (2.19) of Notes on CPM_Sept 2012,

$$\overbrace{\phi(0,\mathbf{a})}^{\phi(kT,\mathbf{a})} = \theta(0,\mathbf{a}) + \theta_0 = 2\pi h\left[a_{-2}q(2T) + a_{-1}q(T)\right] + \theta_0 = 0$$
$$\theta_0 \to \theta_k = -\frac{2\pi}{3} \to \frac{4\pi}{3} \tag{3}$$

The result in (3) agrees with $\theta_k = 4\pi/3$, whilst the initial phase, $\overbrace{\phi(0,\mathbf{a})}^{\phi(kT,\mathbf{a})}$ is zero. The (next) cumulated phase is independent of the present input symbol and is given by (2.5) of Notes on CPM_Sept 2012 as

$$\theta_{k+1} = \theta_k + \pi h a_{k+1-L} = \theta_k + \frac{2\pi}{3}a_{k-2} = \frac{4\pi}{3} + \frac{2\pi}{3} = 2\pi \to 0 \tag{4}$$

Now we are in a position to evaluate the next phase states as follows

$$a_k = +1,\ \text{transition from phase state } [\Phi_{12}\ ,\ 0] \text{ to phase state } [\Phi_4\ ,\ 2\pi/3]$$
$$\phi(kT+T,\mathbf{a}) = \theta(kT+T,\mathbf{a}) + \theta_{k+1} = 2\pi h\left[\overset{1}{a_{k-1}}\,q(2T) + \overset{1}{a_k}\,q(T)\right] + 0 = 2\pi/3$$

$$a_k = -1,\ \text{transition from phase state } [\Phi_{12}\ ,\ 0] \text{ to phase state } [\Phi_2\ ,\ 1.218\pi/3]$$
$$\phi(kT+T,\mathbf{a}) = \theta(kT+T,\mathbf{a}) + \theta_{k+1} = 2\pi h\left[\overbrace{\overset{1}{a_{k-1}}\,q(2T)} + \overbrace{\overset{-1}{a_k}\,q(T)}\right] + 0 = 1.218\pi/3 \tag{5}$$

2) Sample calculations for the transitions from Φ_6 , $3.218\pi/3$ to Φ_9 , $2\pi/3$ and Φ_{11} , $2.782\pi/3$

By setting $a_{-2}\ , a_{-1} = a_{k-2}\ , a_{k-1} = 1,\ -1$, initially we check that

$$\phi(kT,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{-2}q(2T) + a_{-1}q(T)\right] + \theta_k$$
$$= 4\pi/3(0.402 - 0.098) + 2\pi/3 = 3.216\pi/3 \tag{6}$$

(6) means that we can set the initial phase to $3.216\pi/3$.

The (next) cumulated phase is independent of the present input symbol and is given by (2.5) of Notes on CPM_Sept 2012 as

$$\theta_{k+1} = \theta_k + \pi h a_{k+1-L} = \theta_k + \frac{2\pi}{3} a_{k-2} = \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3} \qquad (7)$$

Now we are in a position to evaluate the next phase states as follows

$$a_k = +1, \text{ transition from phase state } [\Phi_6 \ , 3.218\pi/3] \text{ to phase state } [\Phi_{11} \ , 2.782\pi/3]$$

$$\phi(kT+T,\mathbf{a}) = \theta(kT+T,\mathbf{a}) + \theta_{k+1} = 2\pi h \left[\overset{-1}{a_{k-1}} q(2T) + \overset{1}{a_k} q(T) \right] + 4\pi/3 = 2.784\pi/3$$

$$a_k = -1, \text{ transition from phase state } [\Phi_6 \ , 3.218\pi/3] \text{ to phase state } [\Phi_9 \ , 2\pi/3]$$

$$\phi(kT+T,\mathbf{a}) = \theta(kT+T,\mathbf{a}) + \theta_{k+1} = 2\pi h \left[\overbrace{a_{k-1} q(2T)}^{-1} + \overbrace{a_k q(T)}^{-1} \right] + 4\pi/3 = 2\pi/3 \qquad (8)$$

Note that to test (8), we must supply to Matlab file CPMFX_Exp1sur.m, the parameters as follows

initph = '3.218*pi/3*1'; %% $\phi(kT,\mathbf{a}) = 3.218\pi/3$

preH = '[-1 1]' %% Reverse chronological order, a_{-1} , a_{-2} $= -1, \ 1$, instead of a_{-2} , a_{-1} $= 1, \ -1$.

3) The arrangement of the Matlab settings for the transitions from Φ_1 , $4\pi/3$ (first line of Fig 5.2) to Φ_{11} , $2.782\pi/3$ for $a_k = +1$ and Φ_9 , $2\pi/3$ $a_k = -1$.

To implement the first transition of the above, we make he following settings in Matlab file CPMFX_Exp1sur.m

vt = [1 -1]; %To set the input to $a_k = +1$. Note that RHS (the second) input symbol has no effect on the output, since the output slides one column to the right;

initph = '4*pi/3';% To set the initial phase to $\phi(kT,\mathbf{a}) = 4\pi/3$

After pressing run, we obtain the following result in the command window.

phCPM3 = 4.0000 2.7820 % Note that the output is shown without the $\pi/3$ fraction. The first refers to the present phase state of $4\pi/3$, the second indicates the transition to $2.782\pi/3$ due to the input of $a_k = +1$.

Test and verify for the case of $a_k = -1$.