Çankaya University – ECE Department – ECE 632 (FE)

Student Name : Date : 08.01.2013
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Questions

1. (45 Points) Find the phase states in a CPM scheme of 16 states where, $g_T(t) =$ Raised Cosine, $h = 1/2 = 2/4,\ L = 2,\ M = 4$. Draw the sate trellis diagram, identify the distinct states. Verify your results by running CPMFx_Exp1sur.m together with CPMFx_Exp1.mdl available on course wepage.

Solution : According to (2.2) of Notes on CPM_Sept_2012, if the modulation index, h is in the form of $h = 2m/p$ where m and p have no common factors an bearing in mind that a_i can only take on values such as $\mp 1,\ \mp 3,\ \mp 5,\ \cdots \mp (M-1)$ then, the number of phase states due to cumulative phase θ_k is independent of L and given by

$$\theta_k = \frac{2m\pi}{p}\left(\sum_{i=-\infty}^{k-L} a_i\right) = \text{integer multiple of } \frac{2\pi}{p} \qquad (1.1)$$

In our case, $h = 1/2 = 2/4$, then by taking integer multiple as $0,\ 1,\ 2,\ 3$, we obtain the phase states due to cumulative phase as

$$\Phi_{\mathbf{k}}(\theta_k) == \left\{0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}\right\} \qquad (1.2)$$

Note that (1.2) could also be evaluated by using the following recursive relationship and starting from $\theta_{k-1} = 0$

$$\theta_k = \theta_{k-1} + \pi h a_{k-L} \qquad (1.3)$$

Now we include the additional phase states due to M and L to arrive at the total of $N_{\Phi_k} = pM^{L-1} = 16$ number of phase states. For this, we simply modify and use (2.28) of Notes on CPM_Sept_2012 and bear in mind that $q(T) = 0.25$.

Confined to interval $0 \rightarrow 2\pi$

$\Phi_1 : a_{k-1} = -3\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = -0.75\pi \qquad = 5\pi/4$

$\Phi_2 : a_{k-1} = -1\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = -0.25\pi \qquad = 7\pi/4$

$\Phi_3 : a_{k-1} = +1\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = 0.25\pi \qquad = \pi/4$

$\Phi_4 : a_{k-1} = +3\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 0 = 0.75\pi \qquad = 3\pi/4$

$\Phi_5 : a_{k-1} = -3\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi/2 = -0.25\pi \qquad = 7\pi/4$

$\Phi_6 : a_{k-1} = -1\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi/2 = 0.25\pi \qquad = \pi/4$

$\Phi_7 : a_{k-1} = +1\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi/2 = 0.75\pi \qquad = 3\pi/4$

$\Phi_8 : a_{k-1} = +3\ ,\ \theta_k = \pi/2\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi/2 = 1.25\pi \qquad = 5\pi/4$

$\Phi_9 : a_{k-1} = -3\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi = 0.25\pi \qquad = \pi/4$

$\Phi_{10} : a_{k-1} = -1\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi = 0.75\pi \qquad = 3\pi/4$

$\Phi_{11} : a_{k-1} = +1\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi = 1.25\pi \qquad = 5\pi/4$

$\Phi_{12} : a_{k-1} = +3\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + \pi = 1.75\pi \qquad = 7\pi/4$

$\Phi_{13} : a_{k-1} = -3\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 3\pi/2 = 0.75\pi \qquad = 3\pi/4$

$\Phi_{14} : a_{k-1} = -1\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 3\pi/2 = 1.25\pi \qquad = 5\pi/4$

$\Phi_{15} : a_{k-1} = +1\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 3\pi/2 = 1.75\pi \qquad = 7\pi/4$

$\Phi_{16} : a_{k-1} = +3\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h a_{k-1} q(T) + 3\pi/2 = 2.25\pi \qquad = \pi/4$

Distinct phase states $\quad \Phi_k = \left\{ \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4} \right\} \qquad (1.4)$

Test by running CPMFx_Exp1sur.m together with CPMFx_Exp1.mdl proves that the states shown in (1.4) are the correct phase states, provided that an initial phase from amongst the distinct phases is supplied.

The state trellis diagram of (1.4) is shown below.

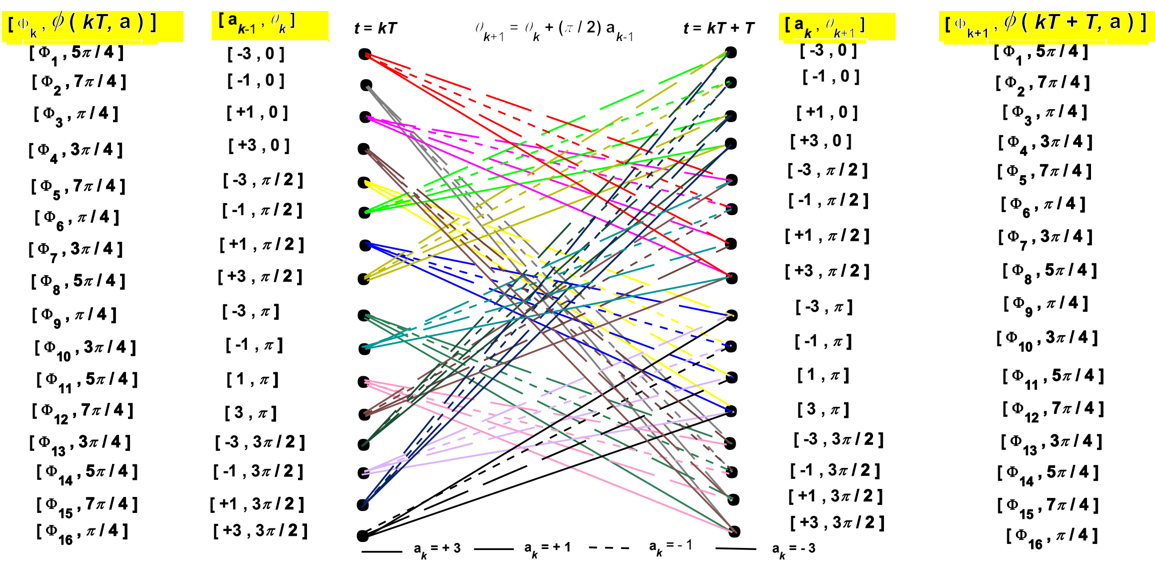

Fig. 1.1 State trellis diagram for $g_T(t) =$ Raised Cosine, $h = 1/2 = 2/4,\ L = 2,\ M = 4$.

2. (25 Points) By running TCM_PeGeneral.m together with TCM_Dec.mdl and TCM_Enc.mdl available on course wepage which delivers the probability error curves of 8 PSK TCM and uncoded 4 PSK, find and plot below which 8 PSK TCM constellations give worse performance than the one arranged according to set partitioning rules of Ungerboeck (stated in ECE 632_Notes on TCM)

Solution : Left as a lab exercise.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) In CPM, there are only four distinct phase states : False, if the modulation index h is real, then the number of phase states is infinite. But if the modulation index h is in the form of $h = 2m / p$, the number of phase states due to accumulated phase is p , including the contribution from M and L (i.e. the instantaneous phase) we get a total of $N_{\Phi_k} = pM^{L-1}$.

b) TCM improves probability of error performance only at high SNR values : True, allow SNR due to nature of coding, there is no improvement, instead worsening of performance.

c) TCM contains some type of coding : True, TCM contains convolutional coding.

d) In CPM, the number of states increases with the modulation index, h : Not exactly, if the modulation index h is in the form of $h = 2m / p$, the number of phase states due to accumulated phase is p .

e) Phase changes in CPM are discrete : If observed only at whole symbol intervals, i.e. at $t = \mp mT$, where m is an integer.

f) Number of phases in CPM increases with M and L : True according to $N_{\Phi_k} = pM^{L-1}$.