Çankaya University – ECE Department – ECE 632 (FE)

Student Name : Date : 06.01.2015
Student Number : Open source exam

Questions

1. (40 Points) Find the phase states in a CPM scheme where, $g_T(t) = \text{Rectangular},\ h = 1/2 = 2/4,\ L = 3,\ M = 2$. Draw the sate trellis diagram, identify the distinct states. Verify your results by running CPMFx_Exp1.m together with CPMFx_Exp1.mdl available on course webpage.

Hint : To evaluate $q(t)$,i.e. the integral of $g_T(t) = \text{Rectangular}$ at $q(2T)$ and $q(T)$ for $L = 3$, you can use (1.4) of Notes on CPM_Sept 2012.

Solution : We start with

$$g_T(t) = \begin{cases} \dfrac{1}{2LT} & 0 \le t \le LT \\ 0 & \text{elsewhere} \end{cases} \tag{1.1}$$

Upon integration we get,

$$q(t) = \int_{-\infty}^{t} g_T(\tau)\, d\tau = \begin{cases} \displaystyle\int_0^t \frac{1}{2LT}\, d\tau = \frac{t}{2LT} & 0 \le t \le T \\ \displaystyle\int_0^T \frac{1}{2LT}\, d\tau = \frac{1}{2L} & t > T \end{cases} \tag{1.2}$$

Thus, with $L = 3$,

$$q(2T) = 1/L = 1/3\ ,\ q(T) = 1/6 \tag{1.3}$$

The number of phase states due to cumulative phase θ_k is independent of L and given by

$$\theta_k = \frac{2m\pi}{p}\left(\sum_{i=-\infty}^{k-L} a_i\right) = \text{integer multiple of } \frac{2\pi}{p} \tag{1.4}$$

In our case, $h = 1/2 = 2/4 = 2m/p,\ p = 4$, then by taking integer multiple as $0,\ 1,\ 2,\ 3$, we obtain the phase states due to cumulative phase as

$$\Phi_{\mathbf{k}}(\theta_k) == \left\{0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}\right\} \tag{1.5}$$

Now we include the additional phase states due to M and L to arrive at the total of $N_{\Phi_k} = pM^{L-1} = 16$ number of phase states. For this, we simply modify and use (2.25) of Notes on CPM_Sept_2012 and use (1.3) above to get

$$\Phi_1 : [a_{k-2}\ ,\ a_{k-1}] = [-1,-1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = 3\pi/2$$

$$\Phi_2 : [a_{k-2}\ ,\ a_{k-1}] = [+1,-1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = \pi/6$$

$$\Phi_3 : [a_{k-2}\ ,\ a_{k-1}] = [-1,+1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = 11\pi/6$$

$$\Phi_4 : [a_{k-2}\ ,\ a_{k-1}] = [+1,+1]\ ,\ \theta_k = 0,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 0 = \pi/2$$

$$\Phi_5 : [a_{k-2}\ ,\ a_{k-1}] = [-1,-1]\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi/2 = 0$$

$$\Phi_6 : [a_{k-2}\ ,\ a_{k-1}] = [+1,-1]\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi/2 = 2\pi/3$$

$$\Phi_7 : [a_{k-2}\ ,\ a_{k-1}] = [-1,+1]\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi/2 = \pi/3$$

$$\Phi_8 : [a_{k-2}\ ,\ a_{k-1}] = [+1,+1]\ ,\ \theta_k = \pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi/2 = \pi$$

$$\Phi_9 : [a_{k-2}\ ,\ a_{k-1}] = [-1,-1]\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi = \pi/2$$

$$\Phi_{10} : [a_{k-2}\ ,\ a_{k-1}] = [+1,-1]\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi = 7\pi/6$$

$$\Phi_{11} : [a_{k-2}\ ,\ a_{k-1}] = [-1,+1]\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi = 5\pi/6$$

$$\Phi_{12} : [a_{k-2}\ ,\ a_{k-1}] = [+1,+1]\ ,\ \theta_k = \pi,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + \pi = 3\pi/2$$

$$\Phi_{13} : [a_{k-2}\ ,\ a_{k-1}] = [-1,-1]\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 3\pi/2 = \pi$$

$$\Phi_{14} : [a_{k-2}\ ,\ a_{k-1}] = [+1,-1]\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 3\pi/2 = 5\pi/3$$

$$\Phi_{15} : [a_{k-2}\ ,\ a_{k-1}] = [-1,+1]\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 3\pi/2 = 4\pi/3$$

$$\Phi_{16} : [a_{k-2}\ ,\ a_{k-1}] = [+1,+1]\ ,\ \theta_k = 3\pi/2,\ \phi(t,\mathbf{a}) = \theta(kT,\mathbf{a}) + \theta_k = 2\pi h\left[a_{k-2}q(2T) + a_{k-1}q(T)\right] + 3\pi/2 = 0$$

Distinct phase states $$\Phi_k = \left\{0,\ \frac{\pi}{6},\ \frac{\pi}{3},\ \frac{\pi}{2},\ \frac{5\pi}{6},\ \pi,\ \frac{7\pi}{6},\ \frac{4\pi}{3},\ \frac{3\pi}{2},\ \frac{11\pi}{6}\right\} \tag{1.6}$$

The state trellis diagram is given in Fig. 1.1.

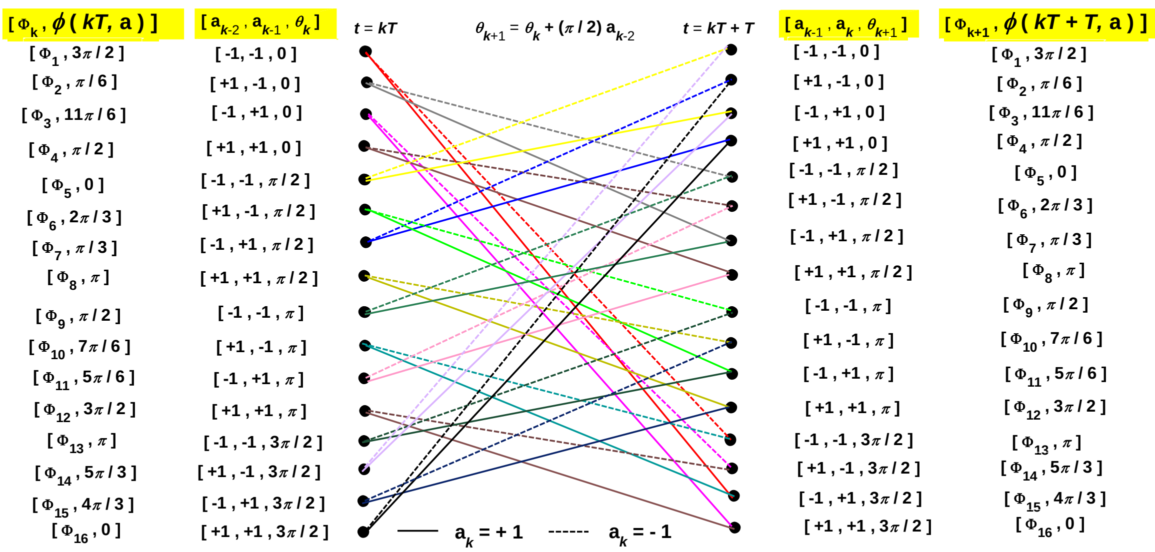

Fig. 1.1 State trellis diagram for $g_T(t) = \text{Rectangular},\ h = 1/2 = 2/4,\ L = 3,\ M = 2$.

2. (30 Points) The convolutional encoder shown below is intended to convert 4 PSK into 8 PSK TCM. Draw the related state trellis diagram. Organize the output (for all possible inputs) in the form of Tables similar to Table 3.1 of ECE 632_Notes on TCM. From there and the state trellis diagram, make an assignment of symbols according to partition rules of Ungerboeck, then draw the constellation diagram of 8 PSK TCM. Compare this constellation diagram with that of Fig. 3.4 of ECE 632_Notes on TCM, commenting on the differences between the two constellations.

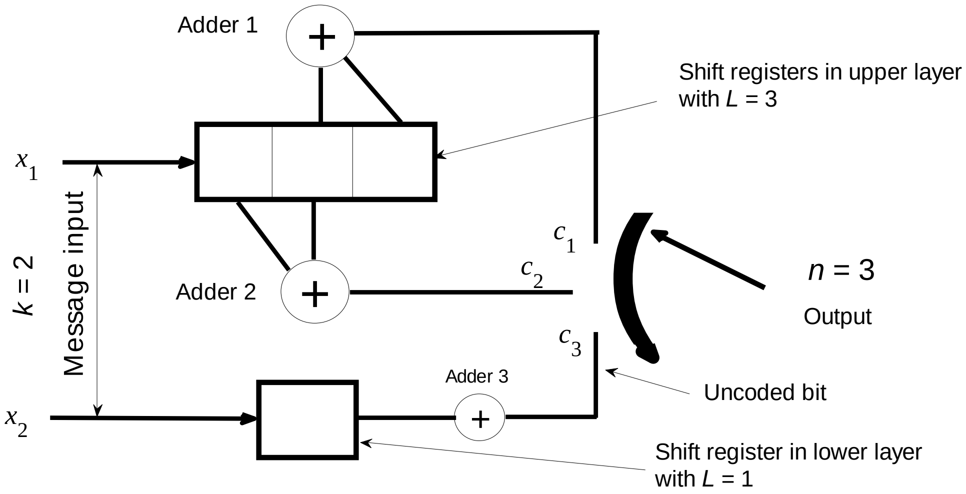

Fig. 2.1 Convolution encoder for Q2.

Solution : By considering Fig. 2.1 and using (2.5) and (2.6) of ECE 632_Notes on TCM

$$\text{Adder 1} = \begin{bmatrix} 0, & 1, & 1 \\ 0, & 0, & 0 \end{bmatrix} \quad \text{Adder 2} = \begin{bmatrix} 1, & 1, & 0 \\ 0, & 0, & 0 \end{bmatrix} \quad \text{Adder 3} = \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 1 \end{bmatrix} \qquad (2.1)$$

We write for the poly2trellis function of Matlab as follows

Constraint length (number of shift registers in upper and lower layers) - L ↓

Connection of Adder 1 to shift registers in upper layer ↓

Connection of Adder 3 to shift registers in upper layer ↓

$$\text{t} = \text{poly2trellis}\big([3,\ 1],\ [3,\ 6,\ \quad 0;0\ 0\ 1]\big)$$

↑ Connection of Adder 2 to shift registers in upper layer

↑ Connection of Adder 3 to shift register in lower layer

(2.2)

Either by hand or by using Conv12_Exp2.m, we get the state trellis diagram drawn in Fig. 2.2. From there we derive Table 2.1 for originating states and Table 2.2 for terminating states.

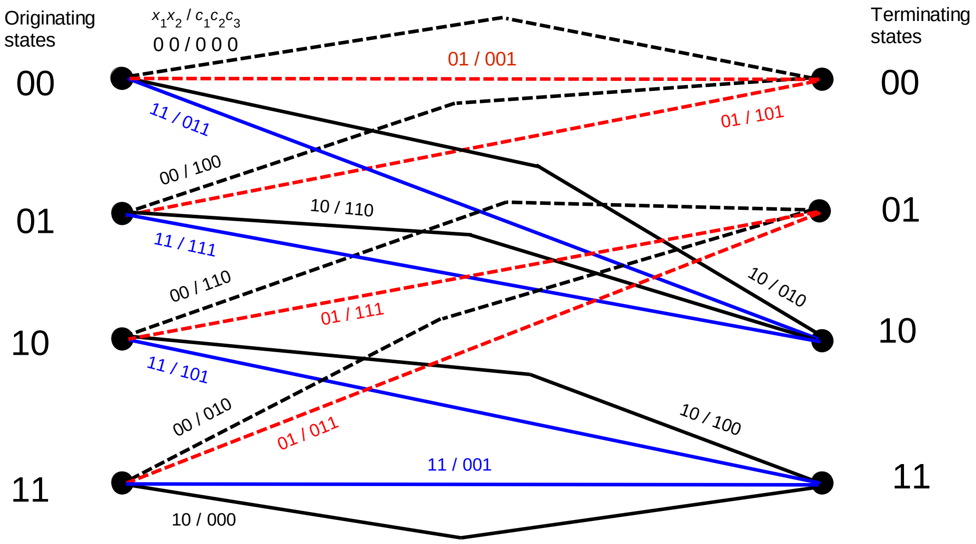

Fig. 2.2 State trellis diagram of the TCM encoder in Fig. 2.1.

Originating state	Input / Output symbol	**s** designation of output sysmbol	**B** designation	Parallel paths
00	00/000	$\mathbf{s}_1$	$\mathbf{B}_{1o}$	Parallel
00	01/001	$\mathbf{s}_2$	$\mathbf{B}_{1o}$	
00	10/010	$\mathbf{s}_3$	$\mathbf{B}_{1o}$	Parallel
00	11/011	$\mathbf{s}_4$	$\mathbf{B}_{1o}$	
01	00/100	$\mathbf{s}_5$	$\mathbf{B}_{2o}$	Parallel
01	01/101	$\mathbf{s}_6$	$\mathbf{B}_{2o}$	
01	10/110	$\mathbf{s}_7$	$\mathbf{B}_{2o}$	Parallel
01	11/111	$\mathbf{s}_8$	$\mathbf{B}_{2o}$	
10	00/110	$\mathbf{s}_7$	$\mathbf{B}_{2o}$	Parallel
10	01/111	$\mathbf{s}_8$	$\mathbf{B}_{2o}$	
10	10/100	$\mathbf{s}_5$	$\mathbf{B}_{2o}$	Parallel
10	11/101	$\mathbf{s}_6$	$\mathbf{B}_{2o}$	
11	00/010	$\mathbf{s}_3$	$\mathbf{B}_{1o}$	Parallel
11	01/011	$\mathbf{s}_4$	$\mathbf{B}_{1o}$	
11	10/000	$\mathbf{s}_1$	$\mathbf{B}_{1o}$	Parallel
11	11/001	$\mathbf{s}_2$	$\mathbf{B}_{1o}$	

Table 2.1 Organization of the originating state information in Fig. 2.2 in the form of a table.

Terminating state	Output symbol	$\mathbf{s}$ designation	B designation	Parallel paths
00	00/000	$\mathbf{s}_1$	$\mathbf{B}_{1t}$	Parallel
00	01/001	$\mathbf{s}_2$	$\mathbf{B}_{1t}$	
00	00/100	$\mathbf{s}_5$	$\mathbf{B}_{1t}$	Parallel
00	01/101	$\mathbf{s}_6$	$\mathbf{B}_{1t}$	
01	00/110	$\mathbf{s}_7$	$\mathbf{B}_{2t}$	Parallel
01	01/111	$\mathbf{s}_8$	$\mathbf{B}_{2t}$	
01	00/010	$\mathbf{s}_3$	$\mathbf{B}_{2t}$	Parallel
01	01/011	$\mathbf{s}_4$	$\mathbf{B}_{2t}$	
10	10/010	$\mathbf{s}_3$	$\mathbf{B}_{2t}$	Parallel
10	11/011	$\mathbf{s}_4$	$\mathbf{B}_{2t}$	
10	10/110	$\mathbf{s}_7$	$\mathbf{B}_{2t}$	Parallel
10	11/111	$\mathbf{s}_8$	$\mathbf{B}_{2t}$	
11	10/100	$\mathbf{s}_5$	$\mathbf{B}_{1t}$	Parallel
11	11/101	$\mathbf{s}_6$	$\mathbf{B}_{1t}$	
11	10/001	$\mathbf{s}_2$	$\mathbf{B}_{1t}$	Parallel
11	11/000	$\mathbf{s}_1$	$\mathbf{B}_{1t}$	

Table 2.2 Organization of the terminating state information in Fig. 2.2 in the form of a table.

From Table 2.1, we have the following $\mathbf{B}$ square designations.

$$\mathbf{B}_{1o} = (000,\ 001,\ 010,\ 011) \quad , \quad \mathbf{B}_{2o} = (100,\ 101,\ 110,\ 111)$$
$$\mathbf{B}_{1o} = (\ \mathbf{s}_1,\ \ \mathbf{s}_2,\ \ \mathbf{s}_3,\ \ \mathbf{s}_4) \quad , \quad \mathbf{B}_{2o} = (\ \mathbf{s}_5,\ \ \mathbf{s}_6,\ \ \mathbf{s}_7,\ \ \mathbf{s}_8) \tag{2.3}$$

whereas, the square designations of Table 2.1, are the followings.

$$\mathbf{B}_{1t} = (000,\ 001,\ 100,\ 101) \quad , \quad \mathbf{B}_{2t} = (010,\ 011,\ 110,\ 111)$$
$$\mathbf{B}_{1t} = (\ \mathbf{s}_1,\ \ \mathbf{s}_2,\ \ \mathbf{s}_5,\ \ \mathbf{s}_6) \quad , \quad \mathbf{B}_{2t} = (\ \mathbf{s}_3,\ \ \mathbf{s}_4,\ \ \mathbf{s}_7,\ \ \mathbf{s}_8) \tag{2.4}$$

Comparing (2.3) to (2.4), we do not find the same $\mathbf{B}$ designations for originating and terminating cases. Recall that in Table 5.1 of ECE 587_Notes on Codes, the same $\mathbf{B}$ designations for originating and terminating cases cloud be made. From these observations, we deduce that it is not possible to apply partitioning rules of Ungerboeck to the encoder of Fig. 2.1 and obtain desirable probability of error performance in 8 PSK TCM, compared with uncoded 4 PSK.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) In CPM, there are only four distinct phase states : False , according to Notes on CPM _Sept 2012, the number of total phase states is given by $N_{\Phi_k} = pM^{L-1}$, where p is related modulation index, M is the Mary level of the ASK input signal, L is the number of symbol duration that the shaping waveform extends over. After finding the state trellis diagram, we can determine the number of distinct phases as shown in (1.6) of Q1.

b) TCM improves probability of error performance only at high SNR values : True, as demonstrated by Exercise 3.3 of ECE 632_Notes on TCM.

c) TCM contains some type of coding : True, TCM involves convolutional coding.

d) In CPM, the number of phase states increases with the modulation index, h : If the modulation index is in the form of $h = 2m/p$, where m and p are integers, then it is possible to assert that In CPM, the number of phase states increases with the denominator of modulation index, h.

e) Phase changes in CPM are discrete : Appears to be as such, if no samples are taken between kT intervals, in reality it is continuous as seen from Figs. 2.1, 2.2 and 2.3 of Notes on CPM_Sept 2012.

f) Number of phase in CPM increase with M and L : True, as seen from $N_{\Phi_k} = pM^{L-1}$.