Çankaya University – ECE Department – ECE 632 (MT)

Student Name : Date : 18.11.2014
Student Number : Open Source Exam

Questions

1. (70 Points) Constellations A and B are as illustrated in Figs. 1.1 and 1.2.

   a. Identify the type of modulation and dimensionality in these constellations. Write and plot the mathematical expression for the basis functions, $\psi_1(t)$, $\psi_2(t)$, write for the signal vectors $\mathbf{s}_1^A \cdots \mathbf{s}_8^A$ and $\mathbf{s}_1^B \cdots \mathbf{s}_8^B$, write and plot the corresponding signal waveforms of $s_1^A(t) \cdots s_8^A(t)$ and $s_1^B(t) \cdots s_8^B(t)$. Find the distance between signal vector ends in both constellations. Determine what the signal vector lengths of B_1 and B_2 in terms of signal vector length A should be such that both constellation A and constellation B use the same total or average energy. Note that the total energy is the sum of squares of vector lengths, whereas the average energy is obtained upon dividing the total energy by the number of symbols.
   b. Draw the demodulator as correlator and matched filter. Assuming that the signal $s_1^A(t)$ from constellation A and the signal $s_1^B(t)$ from constellation B are transmitted, find the outputs of the correlator and matched filter.
   c. Find the probability of error and decision regions via the evaluations of correlation metrics $C(\mathbf{r}, \mathbf{s}_m)$ again assuming $s_1^A(t)$ from constellation A and $s_1^B(t)$ from constellation B were transmitted. Comment on whether you find any probability of error performance difference between constellation A and constellation B.

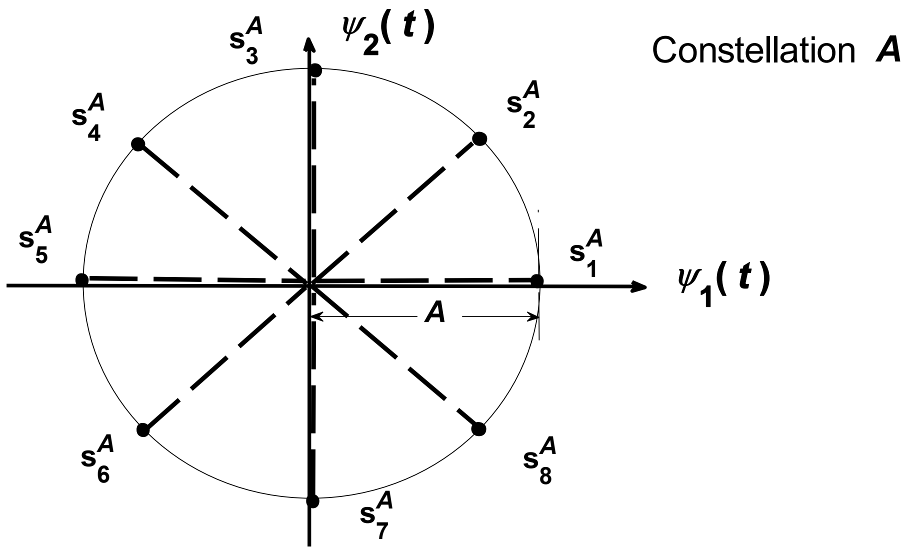


Fig. 1.1 Constellation A.

Note : You can write for the signal vectors $\mathbf{s}_1^A \cdots \mathbf{s}_8^A$ and $\mathbf{s}_1^B \cdots \mathbf{s}_8^B$ in time divided notation or in complex notation.

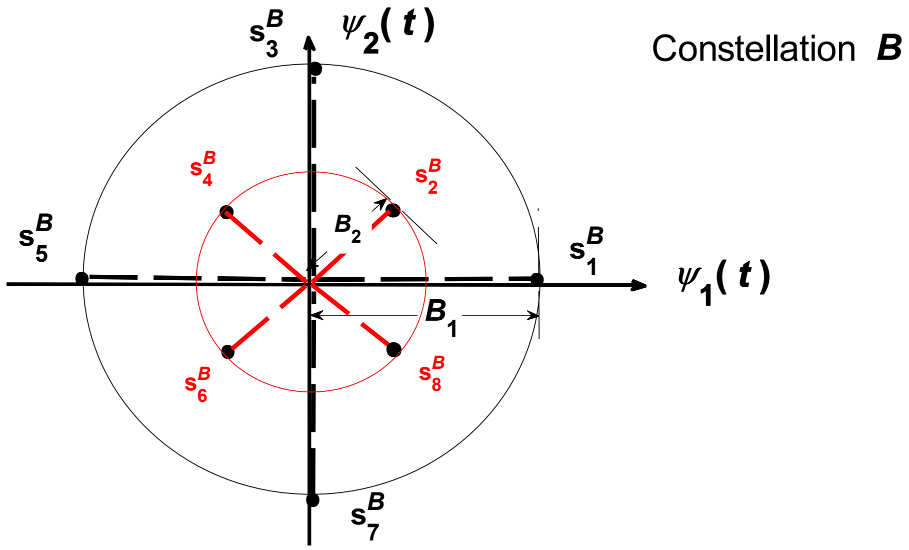


Fig. 1.2 Constellation B.

Solution : a. Constellation A is 8 PSK, constellation B is 8 QAM. Provided that all signals in both constellations are sent from transmitter with equal probability, then equating the total energy in constellation A and B, we get

$$4B_1^2 + 4B_2^2 = 8A^2 \quad , \quad 0.5B_1^2 + 0.5B_2^2 = A^2 \tag{1.1}$$

From (1.1), we see that, there are infinite number of choices for the settings of B_1 and B_2 in relation to A. One possible choice is

$$A = \sqrt{T} \quad , \quad B_1 = 1.25\sqrt{T} \quad , \quad B_2 = 0.6614\sqrt{T} \tag{1.2}$$

We adapt the following common orthonormalized basis functions,

$$\psi_1(t) = \begin{cases} \sqrt{2/T} & 0 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases} \qquad \psi_2(t) = \begin{cases} \sqrt{2/T} & T/2 \le t \le T \\ 0 & \text{otherwise} \end{cases} \tag{1.3}$$

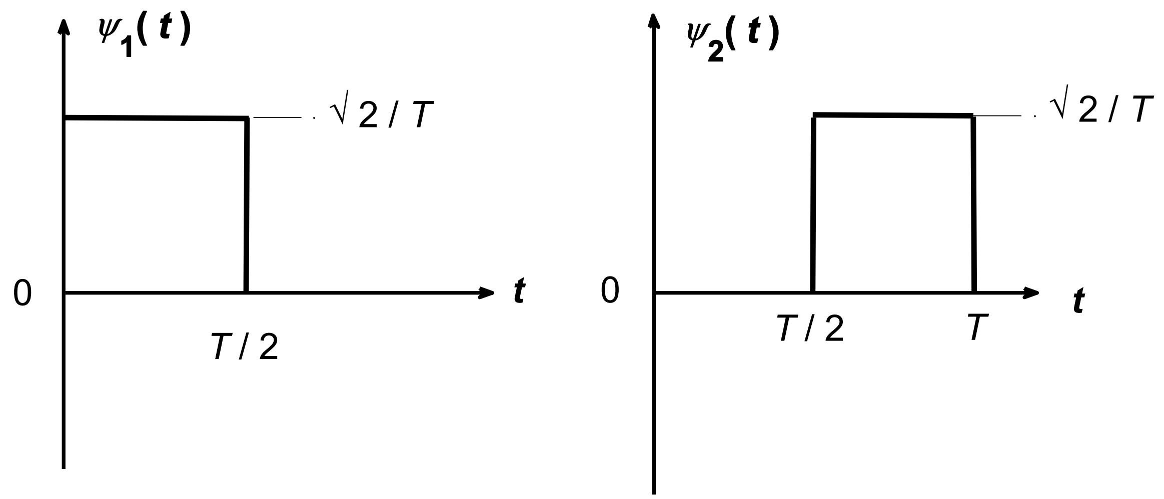


Fig. 1.3 The common orthonormalized basis functions for Q1.

By using Figs. 1.1, 1.3 and (1.3), we obtain the followings for the time waveforms of $s_1^A(t)\cdots s_8^A(t)$

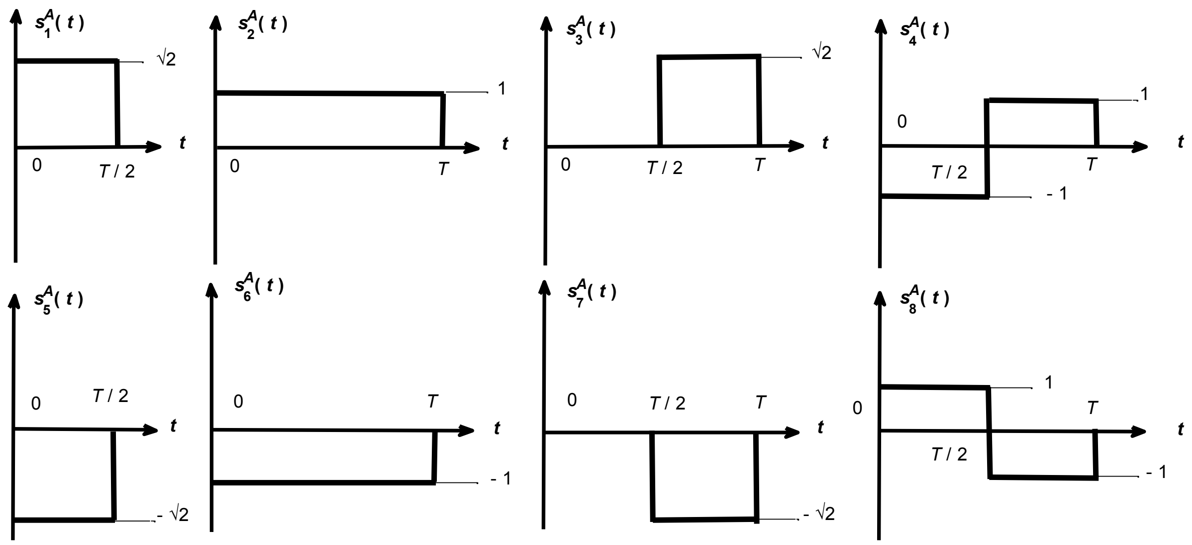


Fig. 1.4 Time waveforms of $s_1^A(t)\cdots s_8^A(t)$ for Q1.

From Fig. 2, it is possible to write the following expressions for $s_1^A(t)\cdots s_8^A(t)$

$$s_1^A(t)=\begin{cases}\sqrt{2} & 0\le t\le T/2\\ 0 & \text{otherwise}\end{cases},\ s_1^A(t)=\sqrt{T}\psi_1(t),\ \mathbf{s_1^A}=\left[s_{11}^A,\ s_{12}^A\right]=\left[\sqrt{T},\ 0\right],\ \varepsilon_{s_1^A}=\left\|\mathbf{s_1^A}\right\|^2=T$$

$$s_2^A(t)=\begin{cases}1 & 0\le t\le T\\ 0 & \text{otherwise}\end{cases},\ s_1^A(t)=\sqrt{\frac{T}{2}}\psi_1(t)+\sqrt{\frac{T}{2}}\psi_2(t),\ \mathbf{s_2^A}=\left[s_{21}^A,\ s_{22}^A\right]=\left[\sqrt{\frac{T}{2}},\ \sqrt{\frac{T}{2}}\right],\ \varepsilon_{s_2^A}=\left\|\mathbf{s_2^A}\right\|^2=T$$

$$s_3^A(t)=\begin{cases}\sqrt{2} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\ s_3^A(t)=\sqrt{T}\psi_2(t),\ \mathbf{s_3^A}=\left[s_{31}^A,\ s_{32}^A\right]=\left[0,\ \sqrt{T}\right],\ \varepsilon_{s_3^A}=T$$

$$s_4^A(t)=\begin{cases}-1 & 0\le t\le T/2\\ 1 & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\ s_1^A(t)=-\sqrt{\frac{T}{2}}\psi_1(t)+\sqrt{\frac{T}{2}}\psi_2(t),\ \mathbf{s_4^A}=\left[s_{41}^A,\ s_{42}^A\right]=\left[-\sqrt{\frac{T}{2}},\ \sqrt{\frac{T}{2}}\right],\ \varepsilon_{s_4^A}=T$$

$$s_5^A(t)=\begin{cases}-\sqrt{2} & 0\le t\le T/2\\ 0 & \text{otherwise}\end{cases},\ s_5^A(t)=-\sqrt{T}\psi_1(t),\ \mathbf{s_5^A}=\left[s_{51}^A,\ s_{52}^A\right]=\left[-\sqrt{T},\ 0\right],\ \varepsilon_{s_5^A}=T$$

$$s_6^A(t)=\begin{cases}-1 & 0\le t\le T\\ 0 & \text{otherwise}\end{cases},\ s_6^A(t)=-\sqrt{\frac{T}{2}}\psi_1(t)-\sqrt{\frac{T}{2}}\psi_2(t),\ \mathbf{s_6^A}=\left[s_{61}^A,\ s_{62}^A\right]=\left[-\sqrt{\frac{T}{2}},\ -\sqrt{\frac{T}{2}}\right],\ \varepsilon_{s_6^A}=T$$

$$s_7^A(t)=\begin{cases}-\sqrt{2} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\ s_7^A(t)=-\sqrt{T}\psi_2(t),\ \mathbf{s_7^A}=\left[s_{71}^A,\ s_{72}^A\right]=\left[0,\ -\sqrt{T}\right],\ \varepsilon_{s_7^A}=T$$

$$s_8^A(t)=\begin{cases}1 & 0\le t\le T/2\\ -1 & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\ s_8^A(t)=\sqrt{\frac{T}{2}}\psi_1(t)-\sqrt{\frac{T}{2}}\psi_2(t),\ \mathbf{s_8^A}=\left[s_{81}^A,\ s_{82}^A\right]=\left[\sqrt{\frac{T}{2}},\ -\sqrt{\frac{T}{2}}\right],\ \varepsilon_{s_8^A}=T \quad (1.4)$$

The time waveforms of $s_1^B(t)\cdots s_8^B(t)$ are shown in Fig. 1.5.

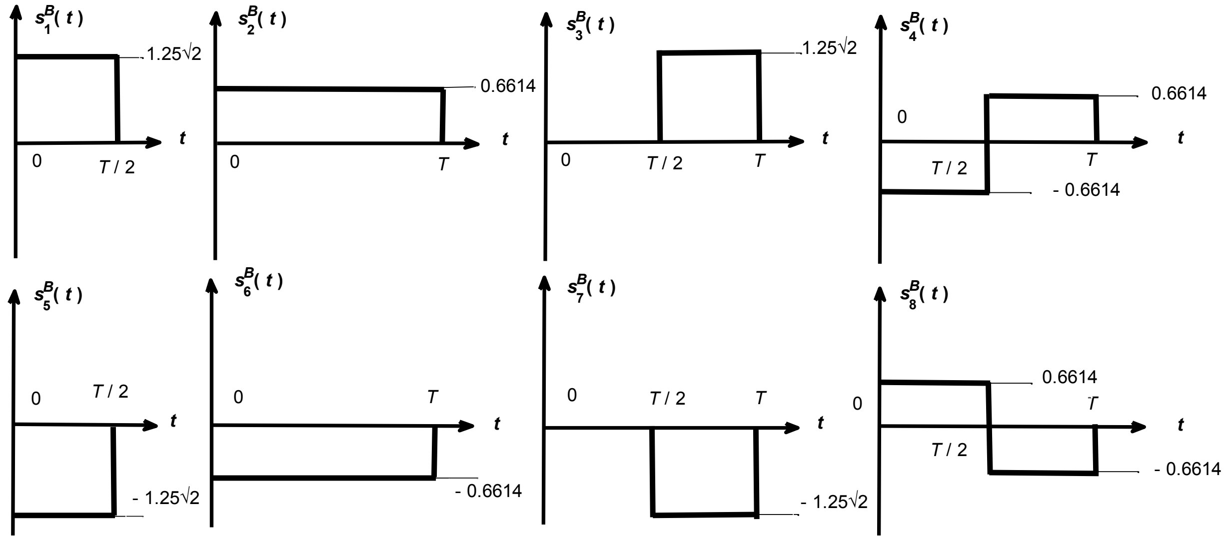


Fig. 1.5 Time waveforms of $s_1^B(t)\cdots s_8^B(t)$ for Q1.

From Fig. 1.5, it is possible to write the following expressions for $s_1^B(t)\cdots s_8^B(t)$

$$s_1^B(t)=\begin{cases}1.25\sqrt{2} & 0\le t\le T/2\\ 0 & \text{otherwise}\end{cases}\ ,\ s_1^B(t)=1.25\sqrt{T}\psi_1(t)\ ,\ \mathbf{s}_1^{\mathbf{B}}=\left[s_{11}^B,\ s_{12}^B\right]=\left[1.25\sqrt{T},\ 0\right]\ ,\ \varepsilon_{s_1^B}=\left\|\mathbf{s}_1^{\mathbf{B}}\right\|^2=1.5625T$$

$$s_2^B(t)=\begin{cases}0.6614 & 0\le t\le T\\ 0 & \text{otherwise}\end{cases}\ ,\ s_2^B(t)=0.4677\sqrt{T}\psi_1(t)+0.4677\sqrt{T}\psi_2(t)$$

$$\mathbf{s}_2^{\mathbf{B}}=\left[s_{21}^B,\ s_{22}^B\right]=\left[0.4677\sqrt{T},\ 0.4677\sqrt{T}\right]\ ,\ \varepsilon_{s_2^B}=\left\|\mathbf{s}_2^{\mathbf{B}}\right\|^2=0.4374T$$

$$s_3^B(t)=\begin{cases}1.25\sqrt{2} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases}\ ,\ s_3^B(t)=1.25\sqrt{T}\psi_2(t)\ ,\ \mathbf{s}_3^{\mathbf{B}}=\left[s_{31}^B,\ s_{32}^B\right]=\left[0,\ 1.25\sqrt{T}\right]\ ,\ \varepsilon_{s_3^B}=1.5625T$$

$$s_4^B(t)=\begin{cases}-0.6614 & 0\le t\le T/2\\ 0.6614 & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases}\ ,\ s_1^B(t)=-0.4677\sqrt{T}\psi_1(t)+0.4677\sqrt{T}\psi_2(t)$$

$$\mathbf{s}_4^{\mathbf{B}}=\left[s_{41}^B,\ s_{42}^B\right]=\left[-0.4677\sqrt{T},\ 0.4677\sqrt{T}\right]\ ,\ \varepsilon_{s_4^B}=0.4374T$$

$$s_5^B(t)=\begin{cases}-1.25\sqrt{2} & 0\le t\le T/2\\ 0 & \text{otherwise}\end{cases}\ ,\ s_5^B(t)=-1.25\sqrt{T}\psi_1(t)\ ,\ \mathbf{s}_5^{\mathbf{B}}=\left[s_{51}^B,\ s_{52}^B\right]=\left[-1.25\sqrt{T},\ 0\right]\ ,\ \varepsilon_{s_5^B}=1.5625T$$

$$s_6^B(t)=\begin{cases}-0.6614 & 0\le t\le T\\ 0 & \text{otherwise}\end{cases}\ ,\ s_6^B(t)=-0.4677\sqrt{T}\psi_1(t)-0.4677\sqrt{T}\psi_2(t)$$

$$\mathbf{s}_6^{\mathbf{B}}=\left[s_{61}^B,\ s_{62}^B\right]=\left[-0.4677\sqrt{T},\ -0.4677\sqrt{T}\right]\ ,\ \varepsilon_{s_6^B}=0.4374T$$

$$s_7^B(t)=\begin{cases}-1.25\sqrt{2} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases}\ ,\ s_7^B(t)=-1.25\sqrt{T}\psi_2(t)\ ,\ \mathbf{s}_7^{\mathbf{B}}=\left[s_{71}^B,\ s_{72}^B\right]=\left[0,\ -1.25\sqrt{T}\right]\ ,\ \varepsilon_{s_7^A}=1.5625T$$

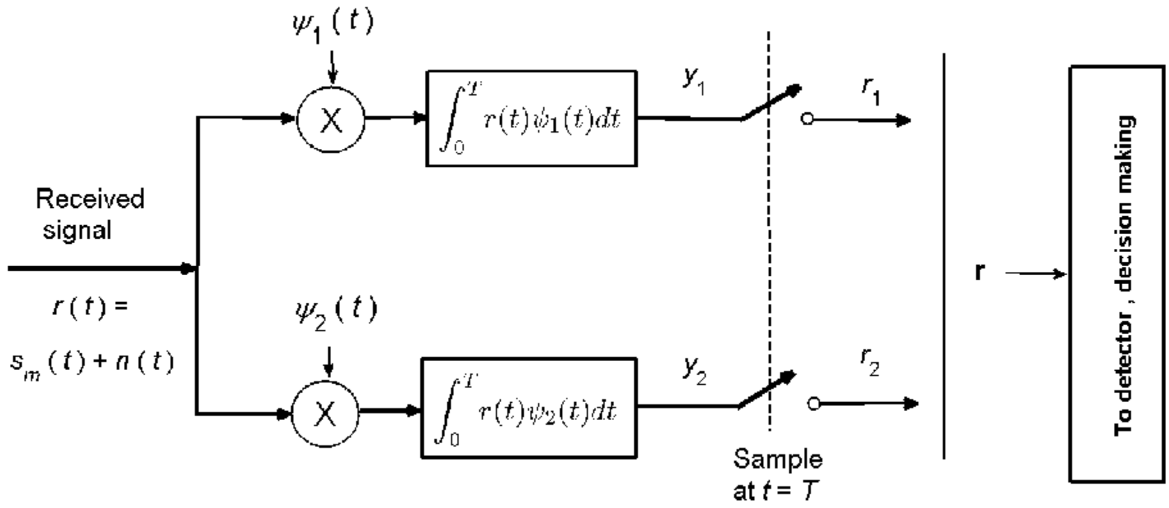
$$s_8^B(t)=\begin{cases}0.6614 & 0\le t\le T/2\\ -0.6614 & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases}\ ,\ s_8^B(t)=0.4677\sqrt{T}\psi_1(t)-0.4677\sqrt{T}\psi_2(t)$$

$$\mathbf{s}_8^{\mathbf{B}}=\left[s_{81}^B,\ s_{82}^B\right]=\left[0.4677\sqrt{T},\ -0.4677\sqrt{T}\right]\ ,\ \varepsilon_{s_8^B}=0.4374T \qquad (1.5)$$

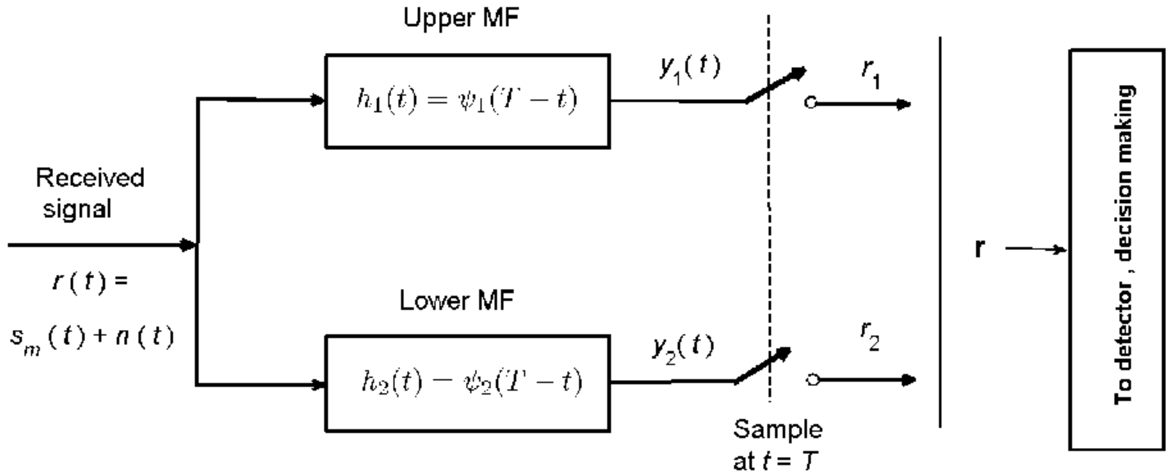
Minimum distance for both constellations are given below.

$$d_{12}^A=d_{\min}^A=0.7654\sqrt{T}\ ,\ d_{12}^B=d_{\min}^B=0.9115\sqrt{T} \qquad (1.6)$$

b. Since QAM and PSK are both two dimensional, for block diagrams of correlator and MF, we benefit from Fig. 6.7 of Notes on Dimensionality of Signals_Sept 2012_HTE, which we reproduce below in Fig. 1.6.



a) Block diagram of correlator type of demodulator.



b) Block diagram of matched filter (MF) type of demodulator.

Fig. 1.6 Block diagrams of correlator and matched filter type of demodulators for the signal set in Q1.

If $s_1^A(t)$ is sent from the transmitter in Constellation A and $s_1^B(t)$ is sent from the transmitter in Constellation B, then the output from the upper and lower braches of the correlator will be (before or after sampling)

$$y_1^A=\int_0^T r(t)\ \psi_1(t)dt=\int_0^T s_1^A(t)\ \psi_1(t)dt+\int_0^T n(t)\ \psi_1(t)dt=s_{11}^A+n_1=\sqrt{T}+n_1$$
$$y_2^A=\int_0^T r(t)\ \psi_2(t)dt=\int_0^T s_1^A(t)\ \psi_2(t)dt+\int_0^T n(t)\ \psi_2(t)dt=s_{12}^A+n_2=n_2$$
$$y_1^B=\int_0^T r(t)\ \psi_1(t)dt=\int_0^T s_1^B(t)\ \psi_1(t)dt+\int_0^T n(t)\ \psi_1(t)dt=s_{11}^B+n_1=1.25\sqrt{T}+n_1$$
$$y_2^B=\int_0^T r(t)\ \psi_2(t)dt=\int_0^T s_1^B(t)\ \psi_2(t)dt+\int_0^T n(t)\ \psi_2(t)dt=s_{12}^B+n_2=n_2 \qquad (1.7)$$

We know that output from MF will be identical to (1.7) at the time of sampling at $t=T$ which means that we can construct the received vector $\mathbf{r}$ that we supply to the detector and which will be used in the decision making process, as follows

$$\mathbf{r^a}=\begin{bmatrix} r_1^a \\ r_2^a \end{bmatrix}=\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix} \quad , \quad \mathbf{r^b}=\begin{bmatrix} r_1^b \\ r_2^b \end{bmatrix}=\begin{bmatrix} 1.25\sqrt{T}+n_1 \\ n_2 \end{bmatrix} \qquad (1.8)$$

d. Using (1.8), we evaluate correlation metrics $C(\mathbf{r},\mathbf{s}_m)$ for $m=1\cdots 8$ as follows

$$C(\mathbf{r^A},\mathbf{s_m^A})=2\,\mathbf{s_m^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_m^A}\right\|^2 \ , \quad m=1\cdots 8$$
$$m=1\ ,\ C(\mathbf{r^A},\mathbf{s_1^A})=2\,\mathbf{s_1^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_1^A}\right\|^2=2\left[\sqrt{T},\ 0\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=T+2n_1\sqrt{T}$$
$$m=2\ ,\ C(\mathbf{r^A},\mathbf{s_2^A})=2\,\mathbf{s_2^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_2^A}\right\|^2=2\left[\sqrt{\frac{T}{2}},\ \sqrt{\frac{T}{2}}\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=\left(\sqrt{2}-1\right)T+2n_1\sqrt{\frac{T}{2}}+2n_2\sqrt{\frac{T}{2}}$$
$$m=3\ ,\ C(\mathbf{r^A},\mathbf{s_3^A})=2\,\mathbf{s_3^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_3^A}\right\|^2=2\left[0,\ \sqrt{T}\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=-T+2n_2\sqrt{T}$$
$$m=4\ ,\ C(\mathbf{r^A},\mathbf{s_4^A})=2\,\mathbf{s_4^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_4^A}\right\|^2=2\left[-\sqrt{\frac{T}{2}},\ \sqrt{\frac{T}{2}}\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=-\left(\sqrt{2}+1\right)T-2n_1\sqrt{\frac{T}{2}}+2n_2\sqrt{\frac{T}{2}}$$
$$m=5\ ,\ C(\mathbf{r^A},\mathbf{s_5^A})=2\,\mathbf{s_5^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_5^A}\right\|^2=2\left[-\sqrt{T},\ 0\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=-3T-2n_1\sqrt{T}$$
$$m=6\ ,\ C(\mathbf{r^A},\mathbf{s_6^A})=2\,\mathbf{s_6^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_6^A}\right\|^2=2\left[-\sqrt{\frac{T}{2}},\ -\sqrt{\frac{T}{2}}\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=-\left(\sqrt{2}+1\right)T-2n_1\sqrt{\frac{T}{2}}-2n_2\sqrt{\frac{T}{2}}$$
$$m=7\ ,\ C(\mathbf{r^A},\mathbf{s_7^A})=2\,\mathbf{s_7^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_7^A}\right\|^2=2\left[0,\ -\sqrt{T}\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=-T-2n_2\sqrt{T}$$
$$m=8\ ,\ C(\mathbf{r^A},\mathbf{s_8^A})=2\,\mathbf{s_8^A}\cdot\mathbf{r^A}-\left\|\mathbf{s_8^A}\right\|^2=2\left[\sqrt{\frac{T}{2}},\ -\sqrt{\frac{T}{2}}\right]\begin{bmatrix} \sqrt{T}+n_1 \\ n_2 \end{bmatrix}-T=\left(\sqrt{2}-1\right)T+2n_1\sqrt{\frac{T}{2}}-2n_2\sqrt{\frac{T}{2}} \qquad (1.9)$$

As detailed in ECE 588MT-19112012_Solutions, to determine the correct decision region for $\mathbf{s_1^A}$, it is sufficient to consider the following ;

$$C(\mathbf{r}^{\mathbf{A}}, \mathbf{s}_1^{\mathbf{A}}) > C(\mathbf{r}^{\mathbf{A}}, \mathbf{s}_2^{\mathbf{A}}) : T + 2n_1\sqrt{T} > (\sqrt{2}-1)T + 2n_1\sqrt{\frac{T}{2}} + 2n_2\sqrt{\frac{T}{2}} \rightarrow (\sqrt{2}-1)(\sqrt{T}+n_1) > n_2$$

$$C(\mathbf{r}^{\mathbf{A}}, \mathbf{s}_1^{\mathbf{A}}) > C(\mathbf{r}^{\mathbf{A}}, \mathbf{s}_8^{\mathbf{A}}) : T + 2n_1\sqrt{T} > (\sqrt{2}-1)T + 2n_1\sqrt{\frac{T}{2}} - 2n_2\sqrt{\frac{T}{2}} \rightarrow (\sqrt{2}-1)(\sqrt{T}+n_1) > -n_2$$

$$C(\mathbf{r}^{\mathbf{A}}, \mathbf{s}_1^{\mathbf{A}}) > C(\mathbf{r}^{\mathbf{A}}, \mathbf{s}_5^{\mathbf{A}}) : T + 2n_1\sqrt{T} > -3T - 2n_1\sqrt{T} \rightarrow \sqrt{T} + n_1 > 0 \tag{1.10}$$

The three conditions in (1.10) define the borders illustrated in Fig. 1.7.

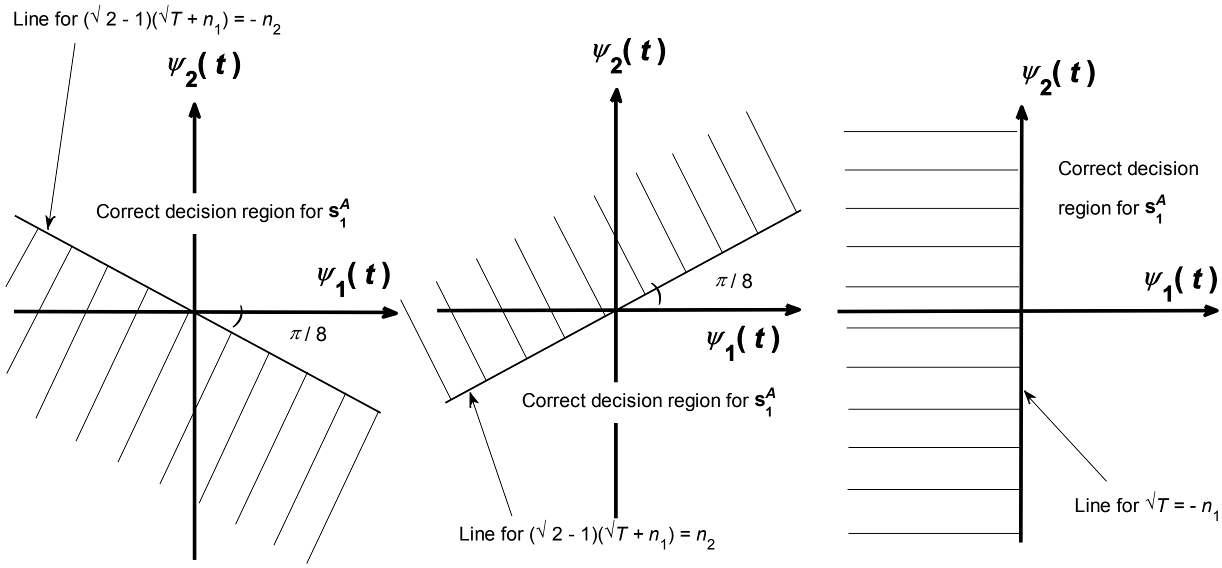


Fig. 1.7 Decision regions for $\mathbf{s}_1^{\mathbf{A}}$.

For probability of error of $\mathbf{s}_1^{\mathbf{A}}$, we benefit from (8) and (9) of SampleProblems of Proakis2002_Nov 2012 and write the following

$$P_c = \frac{1}{(\pi N_0)^{0.5}} \int_{-\sqrt{T}}^{\infty} \exp\left(-\frac{n_1^2}{N_0}\right) dn_1 \frac{1}{(\pi N_0)^{0.5}} \int_{-(\sqrt{T}+n_1)}^{\sqrt{T}+n_1} \exp\left(-\frac{n_2^2}{N_0}\right) dn_2 \quad , \; P_e = 1 - P_c \tag{1.11}$$

Now we repeat the same for $\mathbf{s}_1^{\mathbf{B}}$. For this as stated above, determination of decision borders between $\mathbf{s}_1^{\mathbf{B}}$ and $\mathbf{s}_2^{\mathbf{B}}$, $\mathbf{s}_5^{\mathbf{B}}$, $\mathbf{s}_8^{\mathbf{B}}$ is sufficient. Thus

$$C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_{\mathrm{m}}^{\mathrm{B}}\right) = 2\,\mathbf{s}_{\mathrm{m}}^{\mathrm{B}} \cdot \mathbf{r}^{\mathrm{B}} - \left\|\mathbf{s}_{\mathrm{m}}^{\mathrm{B}}\right\|^2 \;,\quad m = 1,\ 2,\ 5,\ 8$$

$$m=1\;,\; C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_1^{\mathrm{B}}\right) = 2\,\mathbf{s}_1^{\mathrm{B}} \cdot \mathbf{r}^{\mathrm{B}} - \left\|\mathbf{s}_1^{\mathrm{B}}\right\|^2 = 2\left[1.25\sqrt{T},\ 0\right]\begin{bmatrix}1.25\sqrt{T}+n_1\\ n_2\end{bmatrix} - 1.5625T = 1.5625T + 2.5n_1\sqrt{T}$$

$$m=2\;,\; C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_2^{\mathrm{B}}\right) = 2\,\mathbf{s}_2^{\mathrm{B}} \cdot \mathbf{r}^{\mathrm{B}} - \left\|\mathbf{s}_2^{\mathrm{B}}\right\|^2 = 2\left[0.4677\sqrt{T},\ 0.4677\sqrt{T}\right]\begin{bmatrix}1.25\sqrt{T}+n_1\\ n_2\end{bmatrix} - 0.4374T$$

$$= 0.7185T + 0.9354n_1\sqrt{T} + 0.9354n_2\sqrt{T}$$

$$m=5\;,\; C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_5^{\mathrm{B}}\right) = 2\,\mathbf{s}_5^{\mathrm{B}} \cdot \mathbf{r}^{\mathrm{B}} - \left\|\mathbf{s}_5^{\mathrm{B}}\right\|^2 = 2\left[-1.25\sqrt{T},\ 0\right]\begin{bmatrix}1.25\sqrt{T}+n_1\\ n_2\end{bmatrix} - 1.5625T = -4.6875T - 2.5n_1\sqrt{T}$$

$$m=8\;,\; C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_8^{\mathrm{B}}\right) = 2\,\mathbf{s}_8^{\mathrm{B}} \cdot \mathbf{r}^{\mathrm{B}} - \left\|\mathbf{s}_8^{\mathrm{B}}\right\|^2 = 2\left[0.4677\sqrt{T},\ -0.4677\sqrt{T}\right]\begin{bmatrix}1.25\sqrt{T}+n_1\\ n_2\end{bmatrix} - 0.4374T$$

$$= 0.7185T + 0.9354n_1\sqrt{T} - 0.9354n_2\sqrt{T} \qquad (1.12)$$

$$C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_1^{\mathrm{B}}\right) > C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_2^{\mathrm{B}}\right) : 1.5625T + 2.5n_1\sqrt{T} > 0.7185T + 0.9354n_1\sqrt{T} + 0.9354n_2\sqrt{T}$$

$$\rightarrow 0.844\sqrt{T} + 1.5646n_1 > 0.9354n_2 \;\rightarrow\; 1.5646\overbrace{\left(1.25\sqrt{T}+n_1\right)}^{r_1} - 1.11175\sqrt{T} > 0.9354\overbrace{n_2}^{r_2}$$

$$\rightarrow\; r_2 < 1.6727r_1 - 1.1885\sqrt{T}$$

$$C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_1^{\mathrm{B}}\right) > C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_8^{\mathrm{B}}\right) : 1.5625T + 2.5n_1\sqrt{T} > 0.7185T + 0.9354n_1\sqrt{T} - 0.9354n_2\sqrt{T}$$

$$\rightarrow\; r_2 > -1.6727r_1 + 1.1885\sqrt{T}$$

$$C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_1^{\mathrm{B}}\right) > C\left(\mathbf{r}^{\mathrm{B}}, \mathbf{s}_5^{\mathrm{B}}\right) : 1.5625T + 2.5n_1\sqrt{T} > -4.6875T - 2.5n_1\sqrt{T} \;\rightarrow\; r_1 > 0 \qquad (1.13)$$

The three conditions in (1.13) define the borders illustrated in Fig. 1.8.

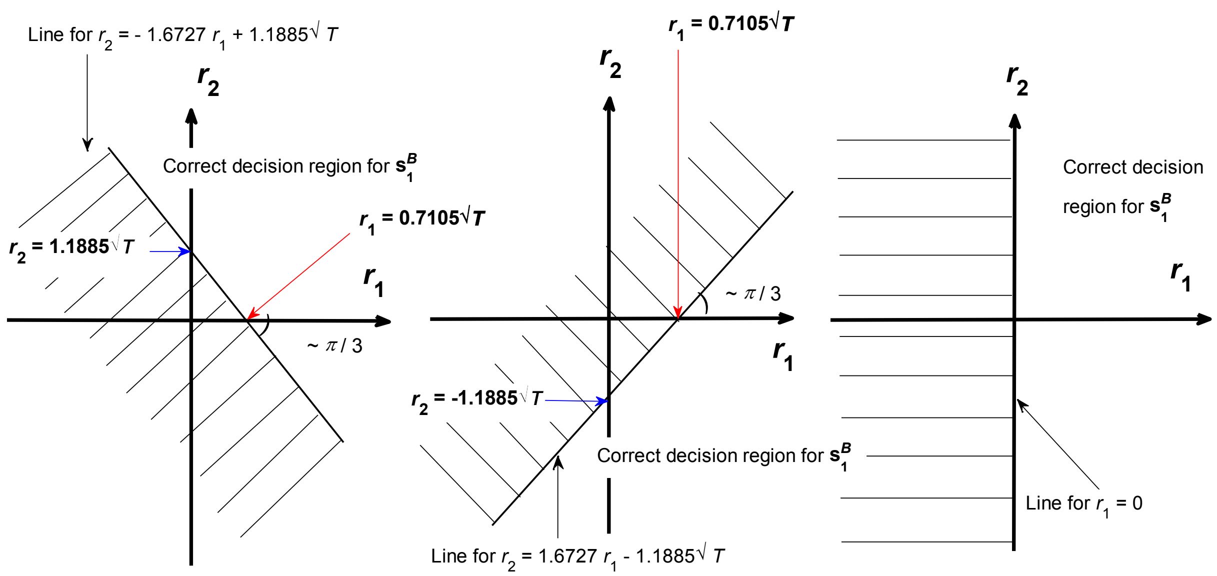


Fig. 1.8 Decision regions for $\mathbf{s}_1^{\mathrm{B}}$.

From the joint borders of Fig. 1.7, we deduce the probability of error for $\mathbf{s}_1^{\mathbf{B}}$ as

$$P_c = \frac{1}{(\pi N_0)^{0.5}} \int_{-0.5395\sqrt{T}}^{\infty} \exp\left(-\frac{n_1^2}{N_0}\right) dn_1 \frac{1}{(\pi N_0)^{0.5}} \int_{-(0.9024\sqrt{T}+1.6727n_1)}^{0.9024\sqrt{T}+1.6727n_1} \exp\left(-\frac{n_2^2}{N_0}\right) dn_2 \quad , \; P_e = 1 - P_c \qquad (1.14)$$

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) QAM offers SNR improvement over PSK : True for $M \geq 8$.

b) The role of the matched filter is to amplify the input signal : False, the role of matched filter is to correlate the input against its time response.

c) In ASK, symbol duration is increased as M increases : This is true for any modulation type, provided of course we do not change the (binary) input rate.

d) FSK is a typical example of a multidimensional signal : True, provided that we have $M > 2$.

e) Optimum detector accepts the optimum signal as input : False, optimum detector attempts to base its decision on maximizing the conditional probability of $P\left(\text{signal } \mathbf{s}_m \text{was transmitted} \middle| \mathrm{r}\right) = P\left(\mathbf{s}_m \middle| \mathrm{r}\right)$

f) The probability of error of higher dimensional signals is lower than lower dimensional signals, but higher dimensional signals require more bandwidth : True, under the conditions same average energy usage, higher dimensional signals have greater minimum distance between the signal vector ends, but since the same time interval is more sliced, higher dimensional signals require more bandwidth.